Observations of Electron Scale Turbulence

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Basic Spatial Electron Scales:

• Spatial scales:
  – Electron gyroradius: \( \rho_e = \frac{v_{\text{perp}}}{\Omega_e} \)
  – Electron inertial length scale \( \lambda_e = \frac{c}{\omega_{pe}} \)
  – Debye length \( \lambda_D = \left( \frac{\varepsilon_0 k_B T_e}{n_e e^2} \right)^{1/2} \)

• Spatial scales (solar wind, foreshock, magnetosheath same order of magnitude):
  – \( \rho_e = 1-2 \times 10^3 \text{ m} \)
  – \( \lambda_e = 1-2 \times 10^3 \text{ m} \)
  – \( \lambda_D = 10 \text{ m} \)
Basic Temporal Electron Scales:

- Temporal scales:
  - Gyrofrequency: \( \Omega_{ce} = e B / m_e \)
  - Electron plasma frequency: \( \omega_{pe} = (n_e e^2 / m_e \varepsilon_0)^{1/2} \)
- Typically values for \( f = \omega / 2\pi \) (solar wind, foreshock, magnetosheath: values same order of magnitude)
  - \( f_{ce} = 2-4 \times 10^2 \) Hz
  - \( f_{pe} = 1-4 \times 10^4 \) Hz
- Scales are Doppler shifted to frequencies in solar wind by: \( \omega = k \cdot v_{\text{flow}} \) if \( v_{\text{phase}} \ll v_{\text{flow}} \)
  - \( f_{pe} = 0.5-1 \times 10^2 \) Hz
  - \( f_{\lambda e} = 0.5-1 \times 10^2 \) Hz
  - \( f_{\lambda D} = 5 \times 10^3 \) Hz
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What is electron scale turbulence?
In this talk: $f \gtrsim 3$ Hz, i.e. $> \text{ion scales}$

Smith et al. 2012
1. Observations: Electron inertial range

\[ 3 \text{ Hz} < f < [f_{\rho e}, f_{\lambda e}] = \sim 50 \text{ Hz} \]
Helios Measurements  (Denskat et al. 1983)

- Magnetic field measurements within solar wind between 0.3 and 1.0 AU
- Helios 2 fluxgate and search coil magnetometer
- Slope of spectra $\sim 1.7$ approx constant within $4 \times 10^{-3}$ Hz to 2 Hz.
- Spectral break around gap between 2 Hz and 4.7 Hz
- Displacement of power spectral density between 2 Hz and 4.7 Hz probably due to damping of Alfven waves near the proton and alpha-particle cyclotron frequencies
- For $f > 4.7$ Hz spectral indicies in the range of $\sim 3$ are observed.
Cluster: Solar wind observations around ion scales (Bale et al. 2005)

- $k \rho_i < 0.45$: magnetic and electric spectrum with $\alpha = -1.7$
- $0.45 < k \rho_i < 2.5$: electric field $\alpha = -1.26$ and magnetic field $\alpha = -2.12$
- $k \rho_i > 2.5$ electric spectrum $\exp(-k \rho_i/12.5)$
- E/B-Ratio fits $v_0(1 + (k \rho_i)^2)$, see red curve in b). This is consistent with kinetic Alfven waves for $k \rho_i$. 

![Graph showing power spectrum and E/B-Ratio fits](image)
Cluster: Magnetosheath up to electron scales
(Alexandrova et al. 2008)

- Spectral knee at ion scales \( \sim 0.3 \) Hz and curvature around 50 Hz; emission of parallel whistler waves at 100 Hz
- Between ion and electron scales \([0.2k \lambda_i, 50 k \lambda_i \approx 1.2k \lambda_e]\):
  - \( B_\perp \) and \( B_\parallel \): \( \alpha = -2.5 \)
Cluster: Foreshock up to electron scales
Sahraoui et al. 2009

- Two breakpoints at 0.5 Hz and 35 Hz.
- Spectral behavior between ion and electron scales
  - $f^{-2.5}$
- Breakpoints fit well with Doppler-shifted: proton and electron gyro-scales

\[ B_{\text{perp}}: \text{FGM (red)} \]
\[ B_{\text{para}}: \text{FGM (black)} \]
\[ B_{\text{perp}}: \text{STAFF-SC burst (blue)} \]
\[ B_{\text{para}}: \text{STAFF-SC burst (green)} \]
Cluster: Foreshock (Sahraoui et al. 2009)

- B scales as $k_{\perp}^{-2.51}$
- E scales as $k_{\perp}^{-0.38}$ (but close to noise level)
- Scaling is consistent with kinetic Alfven waves (KAW)
- Authors show additionally that second break point is consistent with damping of KAW: 
  $\gamma/\omega_r \approx 1$ for $k \rho_e \approx 1$

$V \in [360, 670] \text{km/s}$, $\beta_i \in [0.4, 2]$, $\beta_e \in [0.2, 1.6]$, $\Theta_{BV} \in [65, 85]^\circ$

- Cluster Mag-Observations with FGM, STAFF-SC and STAFF-SA in $f=[1 \times 10^{-3}, 3 \times 10^2] \text{ Hz}$
- Under different plasma conditions the spectrum:
  - $k^{-2.8}$ power law between ion and electron scales
Artemis: Electron density fluctuations
(Chen et al. 2012)

- Slow solar wind
- $\alpha=-2.7$ within $3 < k \rho_i < 15$
- $\alpha=-2.75 \pm 0.06$ from a statistical study of 16 intervals.
Cluster: Solar wind up to electron scales
(Sahraoui et al. 2010)

- Below break point near [0.4,1] $k_{\perp}\rho_i$: steep scaling (transition region): $k_{\perp}^{-4.5}$
- Electron inertial range $k_{\perp}^{-2.8}$
Cluster: Solar wind k-filtering in ion transition region (Sahraoui et al. 2010)

- Transition region (up to 2 Hz, \(~ k_\perp \rho_i \sim 2\))
- Cascade is carried by highly oblique kinetic Alfven waves.
- Observations follow KAW dispersion relationship within \([0.04,2]\) \(k_\perp \rho_i\) (see also Salem et al. 2012)
- The \((k,\omega)\) observations are also in agreement with convected (or slowly propagating) coherent structures with \(k_\perp \gg k_\parallel\) and \(\omega \approx 0\) (Roberts et al. 2013).
Data are consistent with kinetic Alfvén waves with nearly perpendicular wavevectors
Current sheets/discontinuities

Perri et al. 2012

- Cluster observations of thin current sheets in plane perpendicular to \( B \)
- Observed on scales between proton and electron gyroradius
- Might be signs of intermittency and localized areas of turbulent dissipation

\[
\alpha(t, \tau) = \arccos\left(\frac{\mathbf{B}(t) \cdot \mathbf{B}(t + \tau)}{|\mathbf{B}(t)||\mathbf{B}(t + \tau)|}\right).
\]
Anisotropy between ion and electron scales
Chen et al. 2010

- Measurements between $k \rho_i$ and $k \rho_e$
- $(B_\perp / B_\parallel)^2 \approx 0.05$
- Spectral index for $B_\perp$ at small $\Theta$ steepens consistent with critical balanced cascade (KAW or Whistler turbulence)
- Spectral index for $B_\parallel^2$ is less consistent with predictions.
2. Observations: Electron dissipation range

\[ [f_{\rho e}, f_{\lambda e}] = \sim 50 \text{ Hz} \leq f \]

Smith et al. 2012
Electron Dissipation: Power law

- Sahraoui et al. 2009 (Foreshock)
  - $f^{-3.82}$ (electron scales)
  - Breakpoint $\sim$ at Doppler-shifted electron gyro-scales and electron inertial length.

- Sahraoui et al. 2010 (Solar wind)
  - $f^{-3.5}$ (electron scales)
  - Breakpoint $\sim$35 Hz, Doppler-shifted electron gyro-scales $\sim$ 80 Hz.
Statistical study of turbulent spectra to a fraction of $\rho_e$

Alexandrova et al., 2009, 2012

- $E(k_\perp) = E_0 k_\perp^{-8/3} \exp(-k_\perp \rho_e)$ describes well the totality of the observed spectra. $E_0$ is the only free parameter.

- A compensating function $x^{8/3} \exp(x)$, with $x=k_\rho \rho_e$, leads to flat compensated spectra at $k_\rho \rho_e > 0.03$ and for ~2 decades in scales, corresponds to $f=[3,300]$ Hz
Dissipation scale and Universality?

Hydrodynamic turbulence:
Universal Kolmogorov’s function:
\[ E(k) \ell_d / \eta^2 \sim (k \ell_d)^{-5/3} \]

In HD turbulence, this normalization collapses spectra measured under different conditions. Same scaling applied to solar wind spectra and for different candidates for the dissipation scale \( \ell_d \):
\[ \ell_d = \rho_{i,e}, \lambda_{i,e} \]

- Assumption: \( \eta = \text{Const} \)
- \( k \rho_i \) & \( k \lambda_i \) - normalizations are not efficient for collapse
- \( k \rho_e \) normalization bring the spectra close to each other.

[Alexandrova et al., 2009, PRL]
What controls the dissipation Scale?
Alexandrova et al. 2012

• Fitting with the 3 parameter model

\[ E(k_\perp) = A \, k_\perp^{-a} \exp(-k_\perp l_d) \]

\[ l_d = -0.36 + 1.35 \rho_e \]

\[ \text{Cor}=0.70 \]

\[ \text{Cor}=0.34 \]

\( l_d \) is well correlated with \( \rho_e \) confirming the « Kolmogorov Universal function » normalization results, Alexandrova et al. 2009.
Summary

• „Electron inertial scale“: Spectral slopes between ion and electron scales different authors
  – at f>3 Hz, all spectra are quite similar: $\alpha = 2.5$ (foreshock), 2.8 (solar wind) (Sahraoui et al. 2009, 2010, Alexandrova et al. 2009, 2012)
  – Note, that $8/3 \approx 2.6$ is the same as 2.8 when the exp factor is present (Alexandroval et al. 2012)
  – For E and $k_\perp$: $\alpha = 1.36$ (Bale et al. 2005, transition region)
  – Electron density fluctuations show 2.7 spectrum (Chen et al. 2012)
• „Electron dissipation scale“ for $f > f_{\rho_e}$: Spectrum steepenss, but no consensus is reached in the community
  – whether it is universal or not.