Critical Balance

Sean Oughton, Univ of Waikato, New Zealand

Stated assumptions/approx in GoldreichSridhar95

- incompressible MHD with a global B_0
- only shear Alfven wave polarization
- isotropic forcing at scale L

- strong turb, defined as $v_L \sim V_A$ so
  $\varepsilon_{force} = V_A^3 / L$.

  Correct, but misleading: $v_L \to V_A$

- $t_{cas} = 1/(k_{perp}\, v_\lambda)$

  Use k_{perp} since shear polariz?
  Or because assuming perp transfer?

- "critical balance": $t_{cas} = t_{wave}$
- "For the crit bal cascade, interactions are so strong that a `wave packet' lasts for at most a few wave periods" (footnote 2)

Assume a const flux IR:

$$\epsilon_{\text{cas}} = \frac{v_\lambda^2}{t_{\text{cas}}} = k_\perp v_\lambda^3$$

$$= \epsilon_{\text{force}} = \frac{V_A^3}{L} \qquad \Rightarrow \quad v_\lambda \sim V_A (k_\perp L)^{-1/3}$$

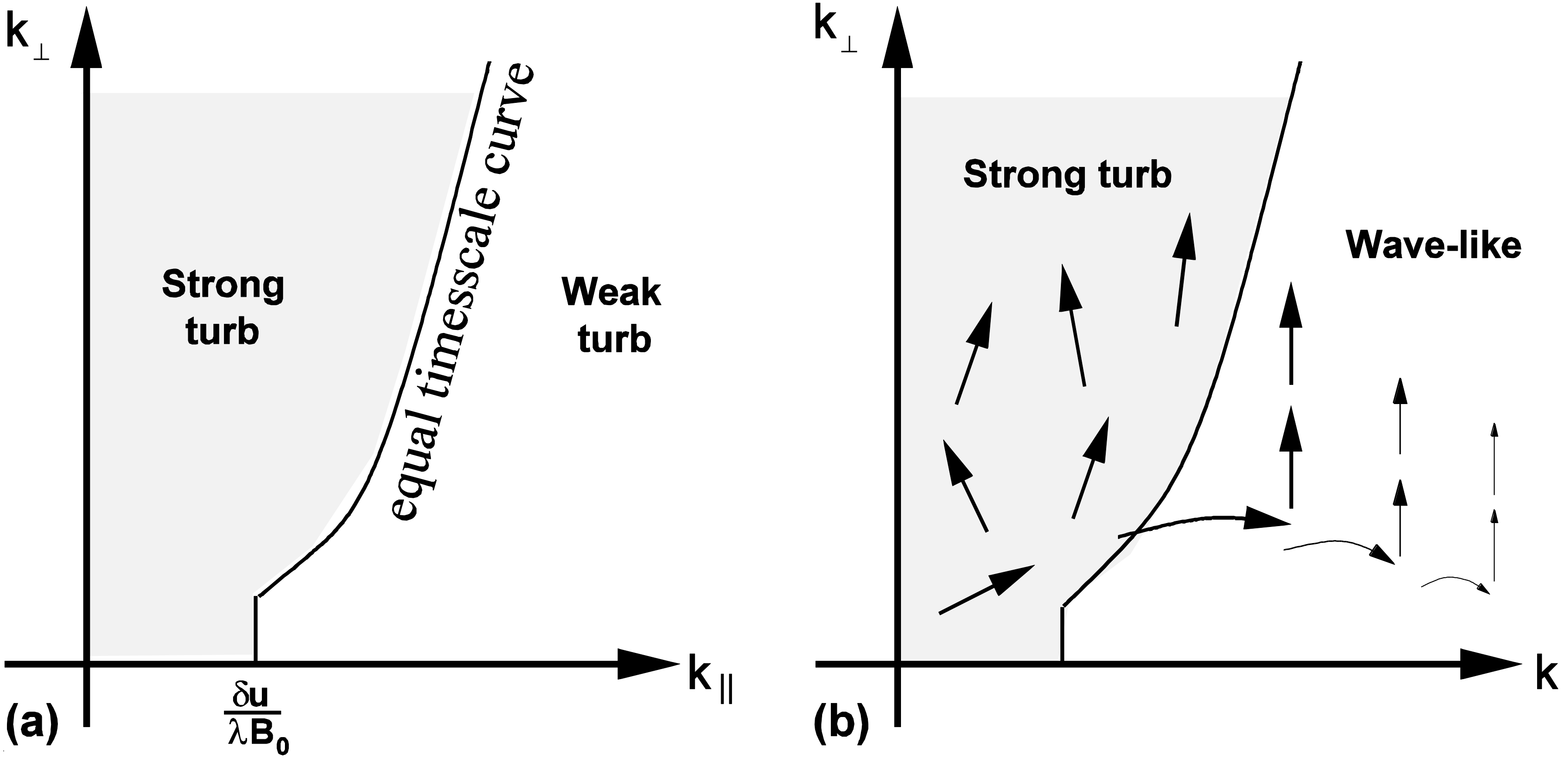
Subst into $t_{cas} = t_{wave}$:
$$k_z \sim k_\perp^{2/3} L^{-1/3} \; \frac{v_L}{V_A}$$

$$\sum_{\text{eddies}} v_\lambda^2 = \int d^3k E(k_\perp, k_z) ,$$

$$E(k_\perp, k_z) \sim \frac{V_A^2}{k_\perp^{10/3} L^{1/3}} f\left(\frac{k_z L^{1/3}}{k_\perp^{2/3}}\right) ,$$

$$f\left(\frac{k_z V_A}{k_\perp v_\lambda}\right)$$

From Oughton et al 2006 Phys Plas and 2013 ApJ



POINT: (almost) always a k-space region where

t_nonlin < t_wave

ie, a strong turb region