The Relative Balance Between Linear and Nonlinear Effects Across the Inertial Range and Ion-Cyclotron Scales

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Solar Wind Turbulence Meeting

Kennebunkport, ME

Jun 7, 2013

What is the relative strengths of linear versus nonlinear wave-wave interactions as measured from direct numerical simulations?

This study

Measure the Linear and Nonlinear forces (accelerations) from direct numerical simulations.

Hall-FLR MHD System

$$\frac{\partial}{\partial t}\rho = -\nabla\cdot(\rho\boldsymbol{u})$$

FLR

$$\frac{\partial}{\partial t}\boldsymbol{u} = -\boldsymbol{u}\cdot\nabla\boldsymbol{u} - \frac{1}{\rho M_{s0}^2}\nabla\cdot[\mathrm{P}\mathbf{I} + \epsilon\boldsymbol{\Pi}] + \frac{\boldsymbol{J}\times\boldsymbol{B}}{\rho M_{a0}^2}$$

$$+\frac{1}{\rho}\nu_o\nabla^2\boldsymbol{u} + \frac{1}{\rho}(\zeta_o + \frac{1}{3}\nu_o)\nabla(\nabla\cdot\boldsymbol{u})$$

$$\frac{\partial}{\partial t}\boldsymbol{A} = \left(\boldsymbol{u} - \epsilon\frac{\boldsymbol{J}}{\rho}\right)\times\boldsymbol{B} - \mu_o\boldsymbol{J} + \nabla\mathrm{F}$$

Hall

$$\Pi_{xx} = 0$$

$$\Pi_{yy} = -\Pi_{zz} = -\frac{\mathrm{P}}{2}[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}]$$

$$\Pi_{yz} = \Pi_{zy} = \frac{\mathrm{P}}{2}[\frac{\partial u_y}{\partial y} - \frac{\partial u_z}{\partial z}]$$

$$\Pi_{zx} = \Pi_{xz} = \mathrm{P}[\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}]$$

$$\Pi_{xy} = \Pi_{yx} = -\mathrm{P}[\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$$

CGL (eg, Yajima, 1966)

$$P = \rho^\gamma/(\gamma M_{s0}^2)\ (\mathrm{isotropic}: P_\perp = P_{||})$$

$$\mathbf{B} = B_0\hat{\mathbf{x}} + \nabla\times\mathbf{A}$$

$$\epsilon = \omega_{\mathrm{A}}/\Omega_i = \omega(k_{\mathrm{min}})/\Omega_i$$

$$\nu \rightarrow \nu(k) = \nu_0[1 + (k/k_{eq})^2]$$

$$\mu \rightarrow \mu(k) = \mu_0[1 + (k/k_{eq})^2]$$

$$F \rightarrow F_k = \frac{i\mathbf{k}\cdot[\mathbf{u}\times\mathbf{B}]_k}{k^2}$$

Wave-wave interactions addressed accurately; wave-particle interactions are absent.

Hall-FLR MHD System

$$\frac{\partial}{\partial t}\boldsymbol{u} = \boldsymbol{M}_{\mathrm{Lin}} + \boldsymbol{M}_{\mathrm{NL}} + \boldsymbol{M}_{\mathrm{diss}}$$

$$\frac{\partial}{\partial t}\boldsymbol{B} = \boldsymbol{N}_{\mathrm{Lin}} + \boldsymbol{N}_{\mathrm{NL}} + \boldsymbol{N}_{\mathrm{diss}}$$

Hall-FLR MHD System

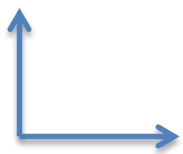
$$\frac{\partial}{\partial t}\boldsymbol{u} = \boldsymbol{M}_{\mathrm{Lin}} + \boldsymbol{M}_{\mathrm{NL}} + \boldsymbol{M}_{\mathrm{diss}} \qquad \mathrm{R_M} = \frac{<|\mathbf{M}_{\mathrm{NL}}|>}{<|\mathbf{M}_{\mathrm{Lin}}|>}$$

$$\frac{\partial}{\partial t}\boldsymbol{B} = \boldsymbol{N}_{\mathrm{Lin}} + \boldsymbol{N}_{\mathrm{NL}} + \boldsymbol{N}_{\mathrm{diss}} \qquad \mathrm{R_N} = \frac{<|\mathbf{N}_{\mathrm{NL}}|>}{<|\mathbf{N}_{\mathrm{Lin}}|>}$$

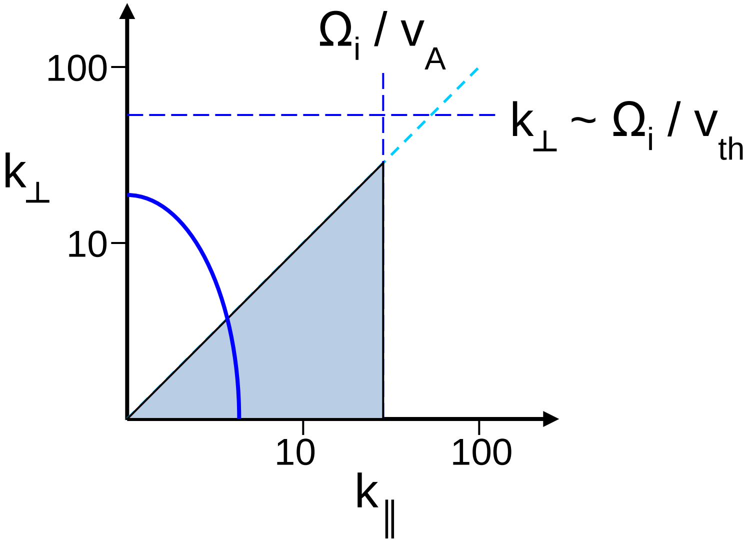
$\mathrm{R_{M,N}} \ll 1 \rightarrow$ Linear dynamics

$\mathrm{R_{M,N}} \gg 1 \rightarrow$ Nonlinear dynamics

Simulations

Momentum $R_M = \dfrac{< \lvert \mathbf{M}_{NL} \rvert >}{< \lvert \mathbf{M}_{Lin} \rvert >}$ 	Mag Induction $R_N = \dfrac{< \lvert \mathbf{N}_{NL} \rvert >}{< \lvert \mathbf{N}_{Lin} \rvert >}$
Momentum $\cos\theta_M = \dfrac{\boldsymbol{M}_{Lin} \cdot \boldsymbol{M}_{NL}}{\lvert \boldsymbol{M}_{Lin} \rvert \lvert \boldsymbol{M}_{NL} \rvert}$	Mag Induction $\cos\theta_N = \dfrac{\boldsymbol{N}_{Lin} \cdot \boldsymbol{N}_{NL}}{\lvert \boldsymbol{N}_{Lin} \rvert \lvert \boldsymbol{N}_{NL} \rvert}$

Simulation Details



- 2 ½-D
- 256x256 resolution (k_{max} = 128)
- Dissipation scale: k ~ 50
- Hall scale: $k_\varepsilon = \Omega_i/V_A = 20$
- FLR scale: $k_L = \Omega_i/v_{th} = \Omega_i/(\beta^{1/2}V_A) = 10 .. 20 .. 40$

Time-Lapse Simulations: VS+Slab

hall51r.avi

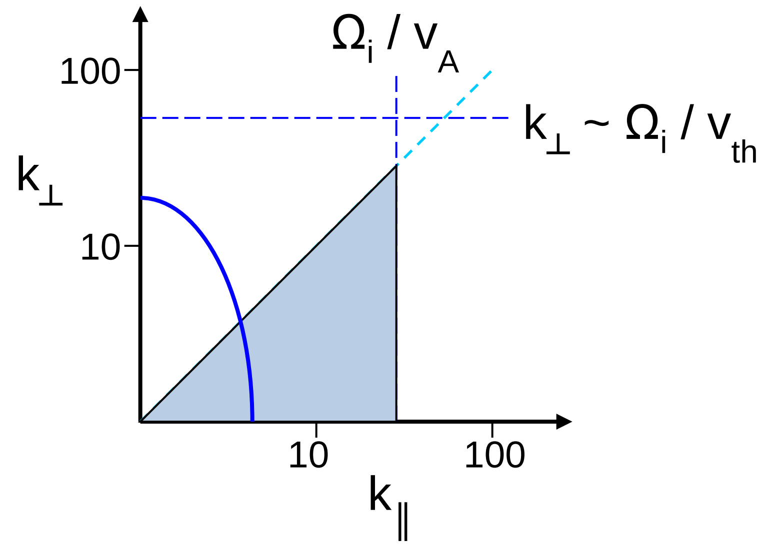
hllflr50r.avi

NL/Lin analysis:

$R_M(k_x,k_y)$ & $R_N(k_x,k_y)$

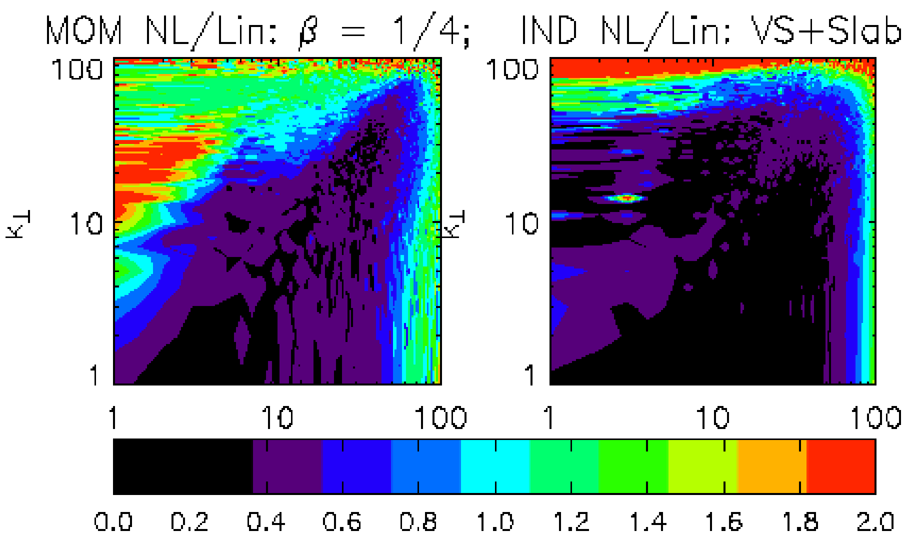
$R_M, R_N << 1 \rightarrow$ Linear

$R_M, R_N \geq 1 \rightarrow$ NL

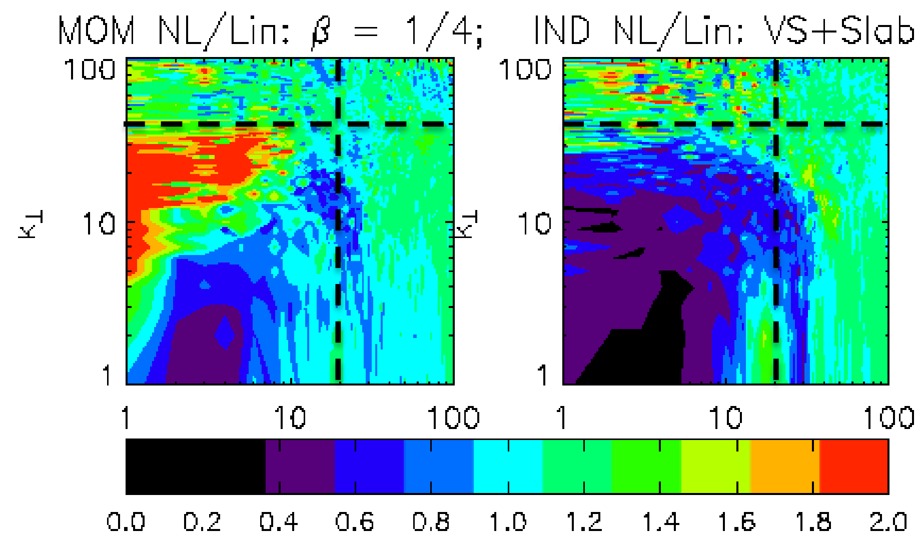


VS+Slab: β = ¼

Standard MHD



Hall-FLR MHD



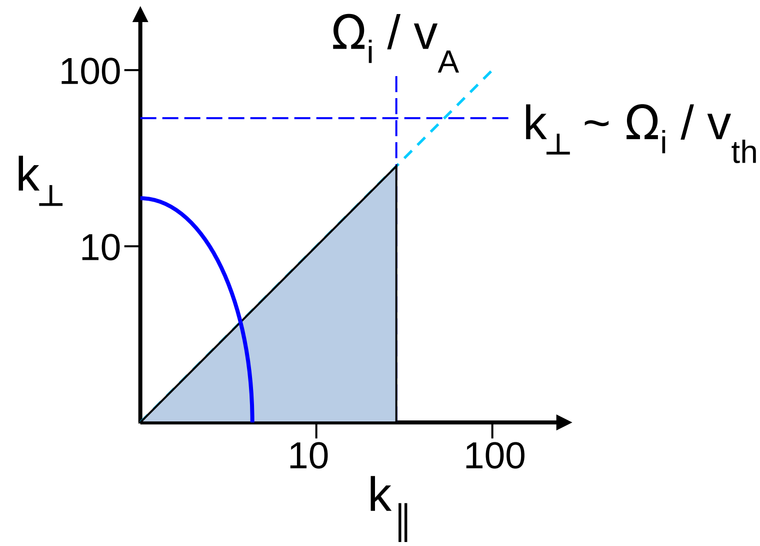
NL/Lin analysis:

$R_M(k_x,k_y)$ & $R_N(k_x,k_y)$

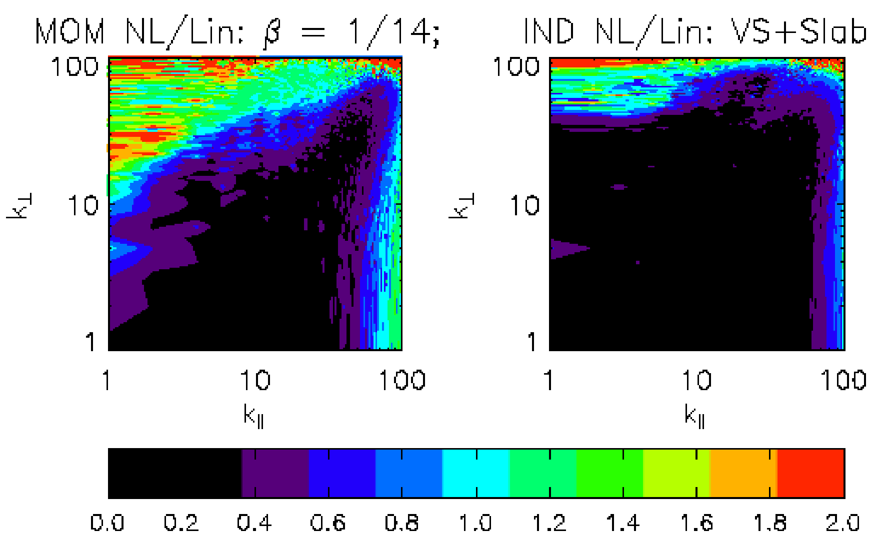
$R_M, R_N << 1 \rightarrow$ Linear

$R_M, R_N \geq 1 \rightarrow$ NL

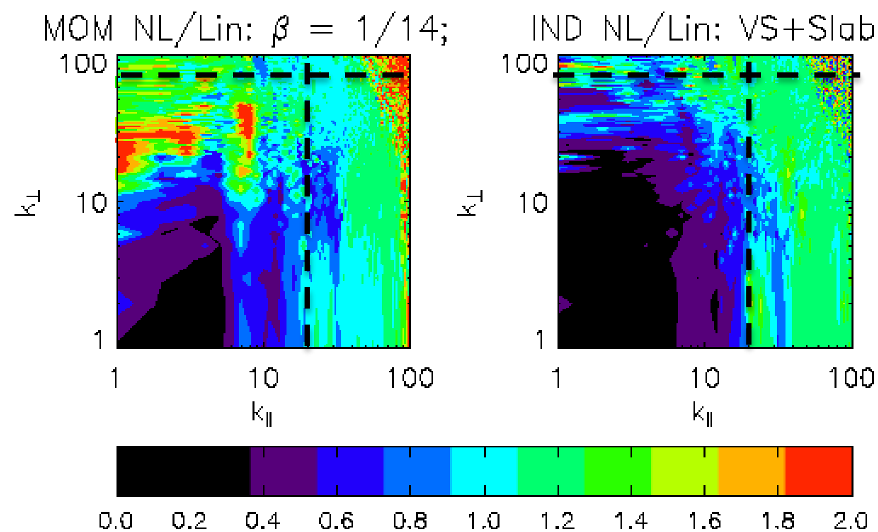
VS+Slab: $\beta = {}^{1}/_{14}$



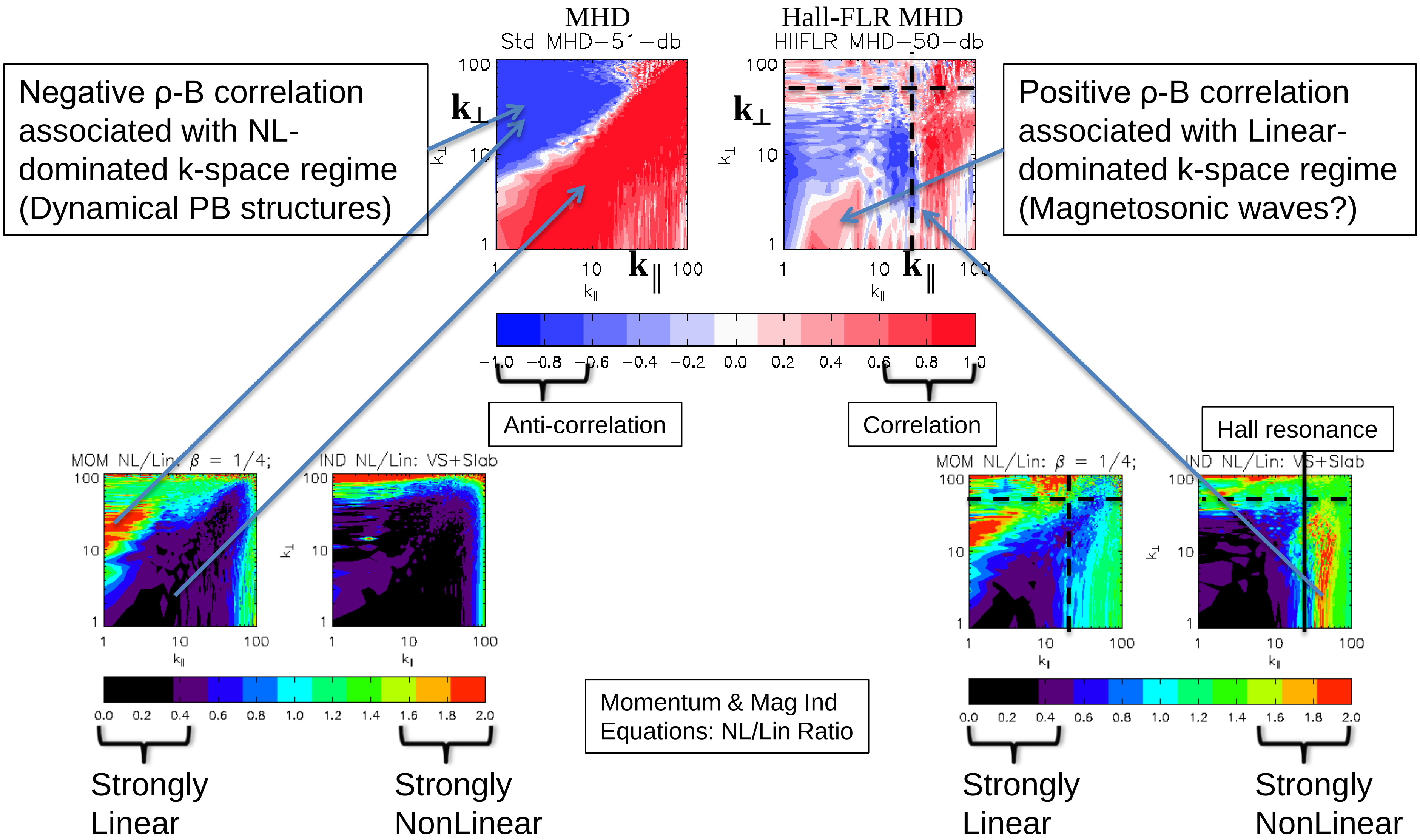
Standard MHD



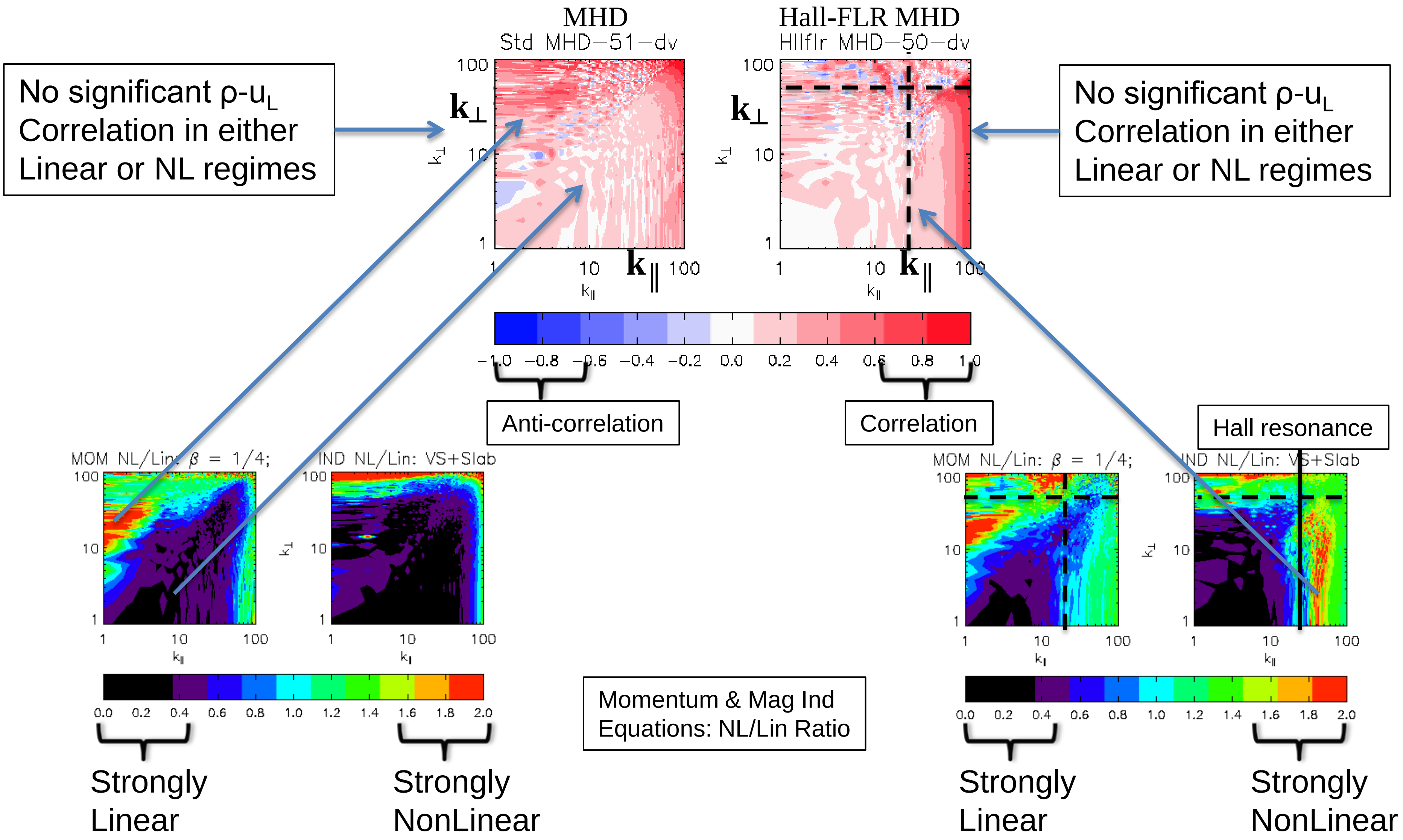
Hall-FLR MHD



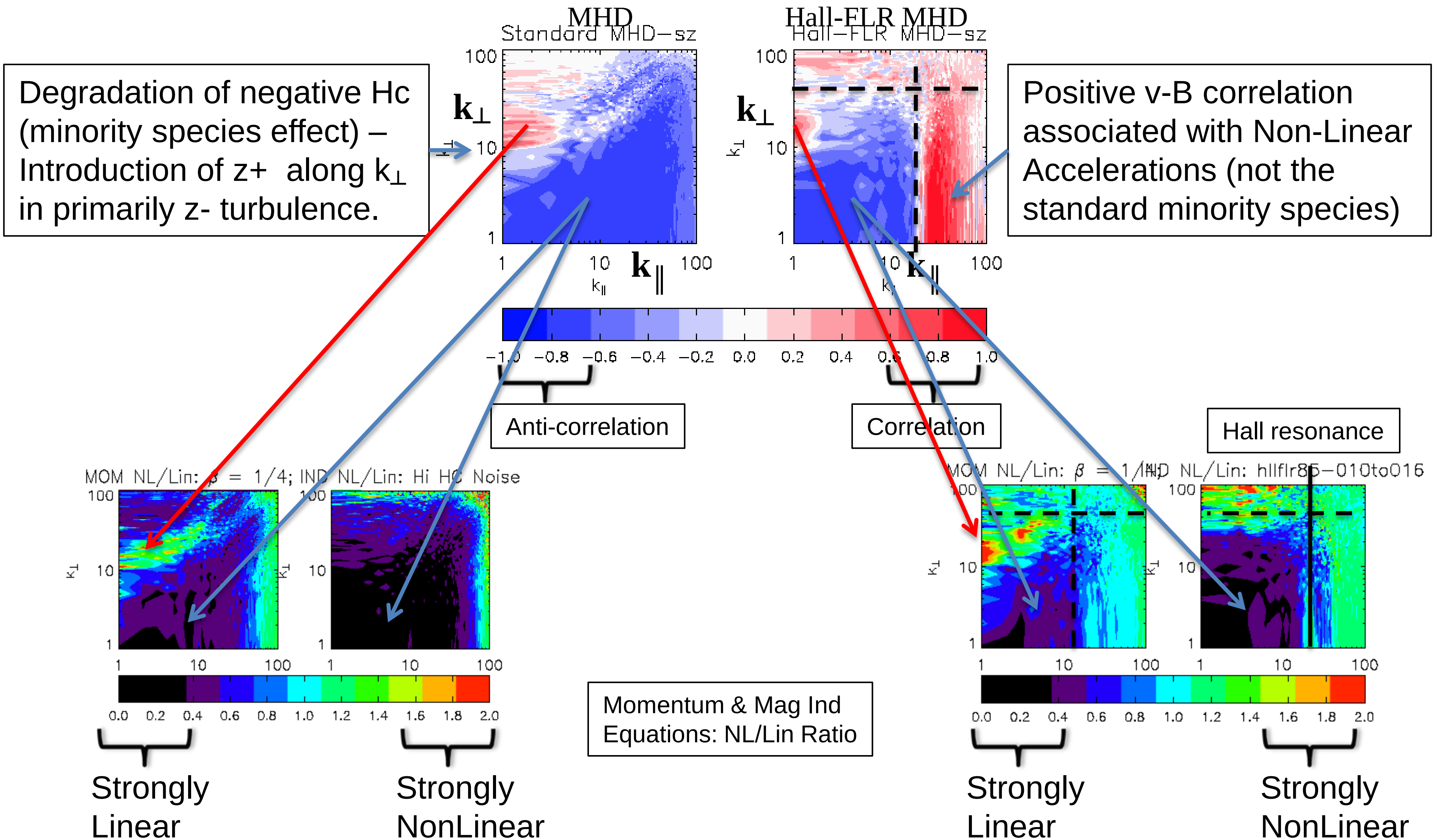
Density-Magnetic Field Correlations



Density-Longitudinal Velocity Correlations

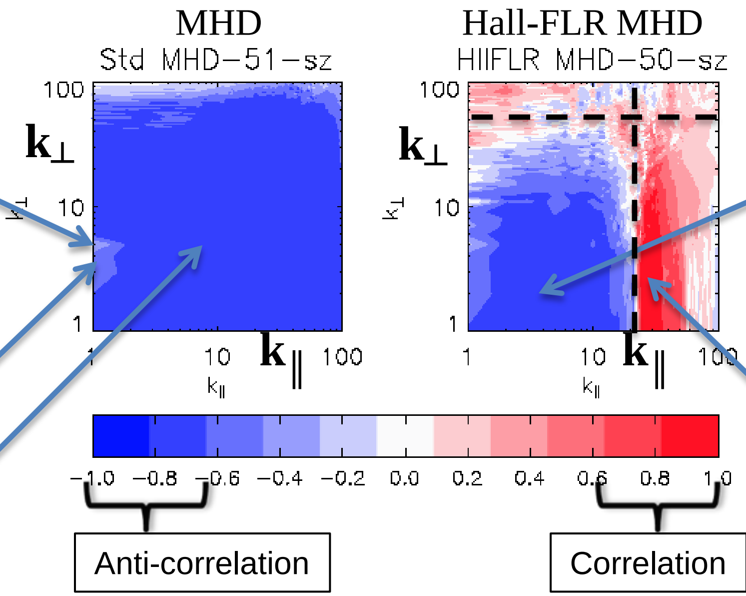


Velocity-Magnetic Field (Cross-Helicity) Correlations – Early Times

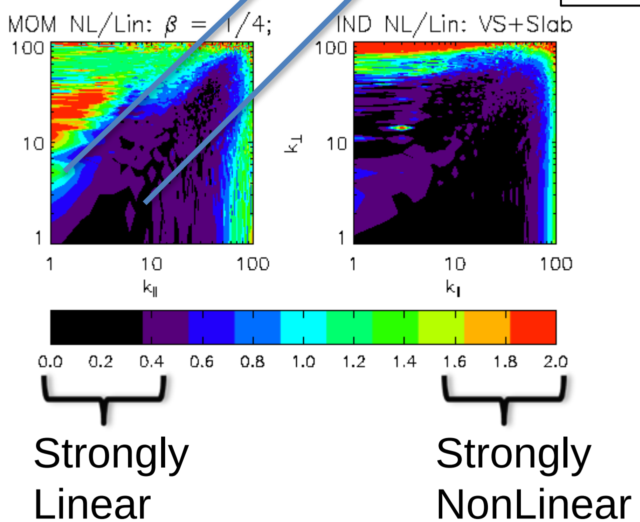


Velocity-Magnetic Field (Cross-Helicity) Correlations

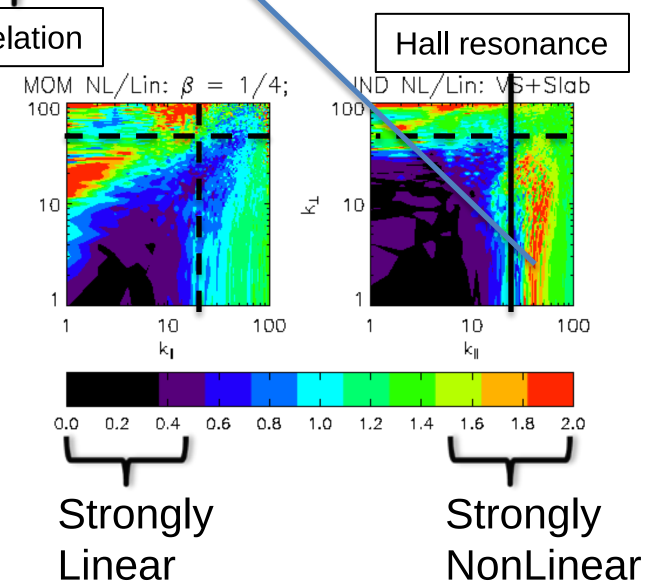
Degradation of negative Hc (minority species effect) – Introduction of z+ along $k_\perp$ in primarily z- turbulence.



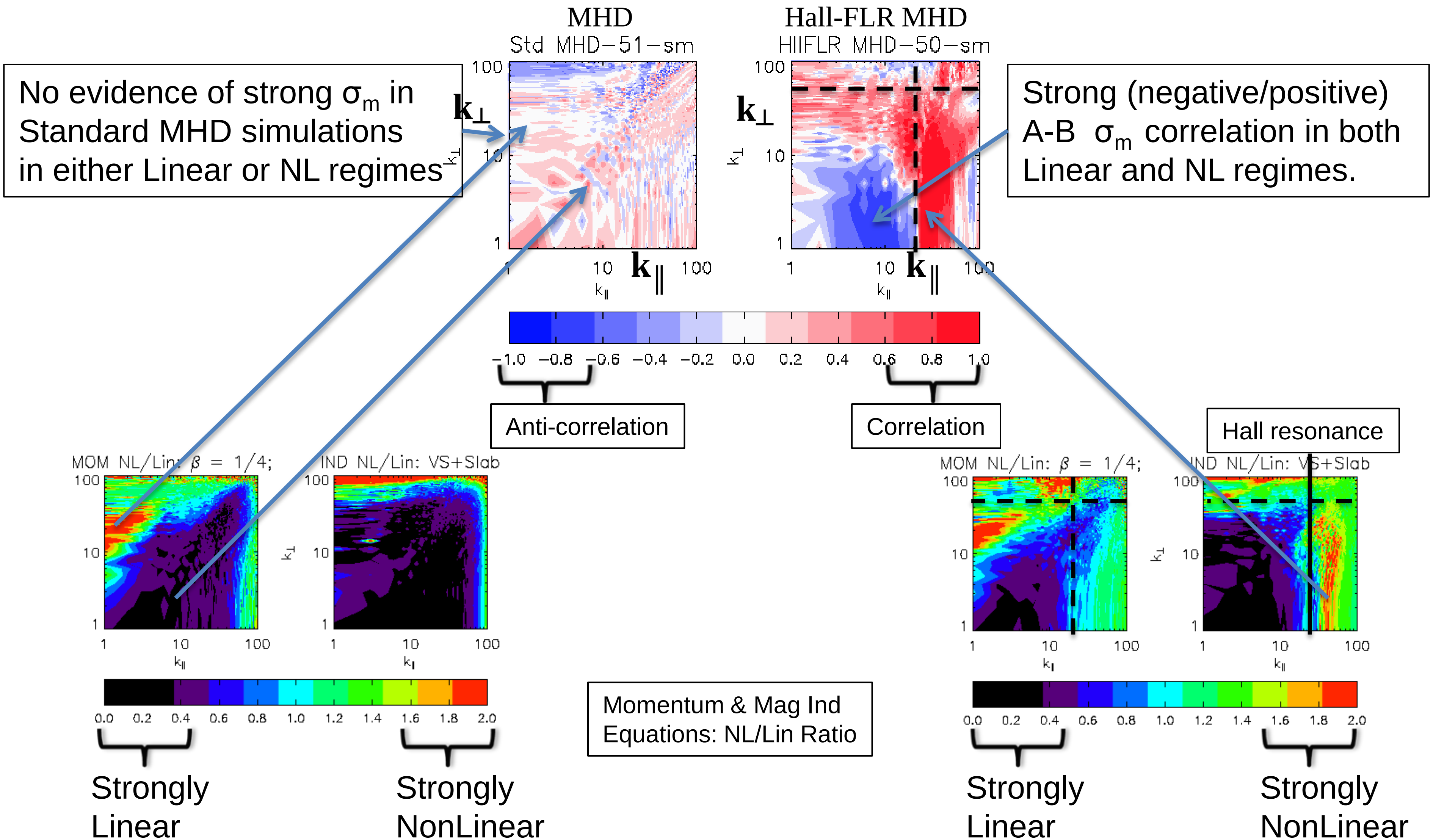
Positive v-B correlation associated with Non-Linear Accelerations (not the standard minority species)



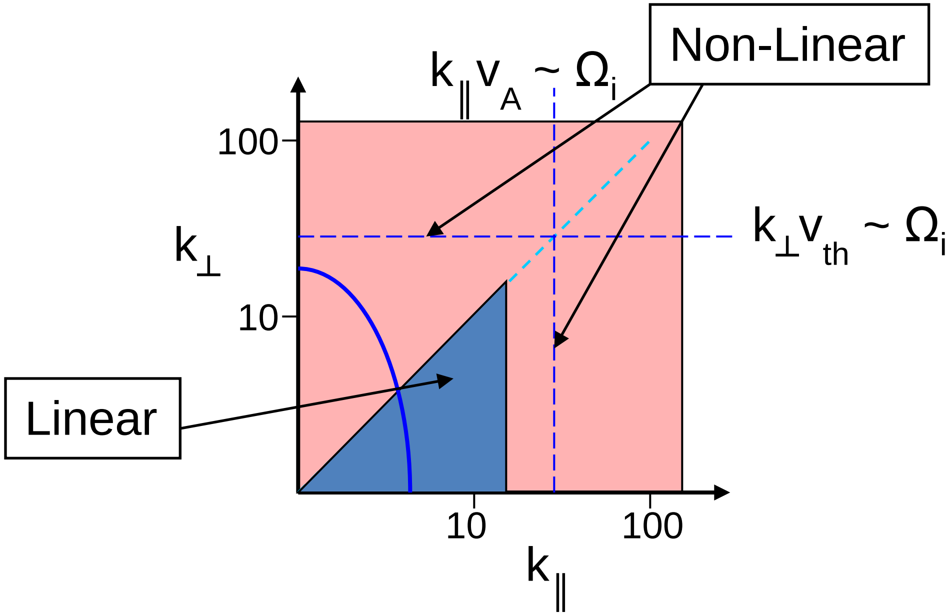
Momentum & Mag Ind Equations: NL/Lin Ratio



Vector Potential-Magnetic Field (Magnetic Helicity) Correlations

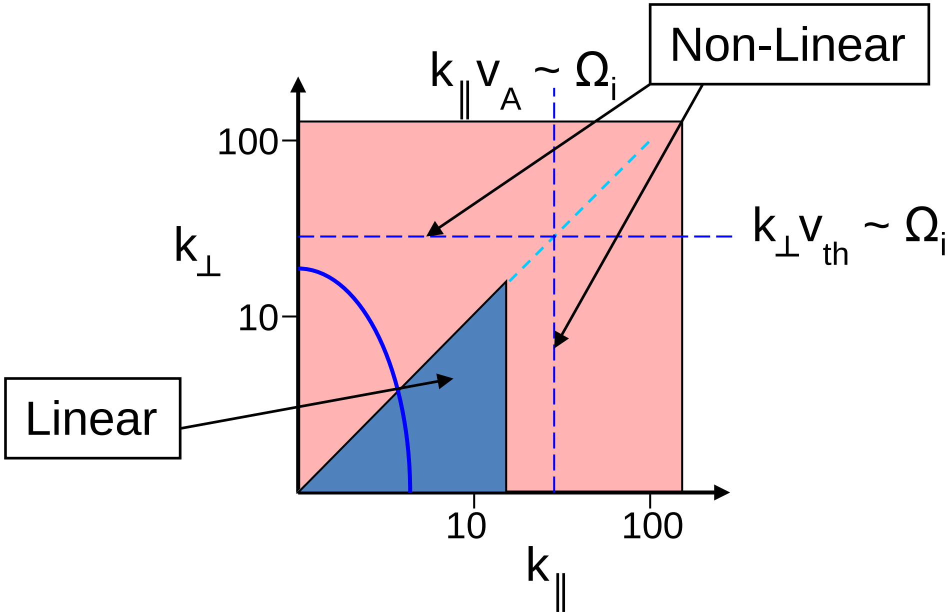


Conclusions



- Non-linear forces dominate across a broad region of k-space – especially at angles $\theta > 45°$ to $\boldsymbol{B}_0$.
- Linear forces persist in a limited region of k-space within angles $\theta < 45°$ to $\boldsymbol{B}_0$.
- Dynamics at Hall-FLR scales appear governed by strongly nonlinear influences.
- Hence, linear treatments (e.g., KAW) may not be appropriate adjacent the dissipation scales in the Solar Wind.

Conclusions (cont'd)



- Correlations (ρ-B , σ_z , σ_m) adjacent the Hall-FLR / dissipation scales may be governed by strongly nonlinear influences; linear theory may not be applicable.
- Disclaimer:
  - Fluid treatment may not apply at ion-cyclotron and smaller scales.