

The Relative Balance Between Linear and Nonlinear Effects Across the Inertial Range and Ion-Cyclotron Scales

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What is the relative strengths of linear versus nonlinear wave-wave interactions as measured from direct numerical simulations?

This study

Measure the Linear and Nonlinear forces (accelerations) from direct numerical simulations.

Hall-FLR MHD System

$$\frac{\partial}{\partial t} \rho = -\nabla \cdot (\rho \mathbf{u})$$

$$\frac{\partial}{\partial t} \mathbf{u} = -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{\rho M_{s0}^2} \nabla \cdot [\mathbf{P}\mathbf{I} + \epsilon \mathbf{\Pi}] + \frac{\mathbf{J} \times \mathbf{B}}{\rho M_{a0}^2}$$

$$+ \frac{1}{\rho} \nu_o \nabla^2 \mathbf{u} + \frac{1}{\rho} (\zeta_o + \frac{1}{3} \nu_o) \nabla (\nabla \cdot \mathbf{u})$$

$$\frac{\partial}{\partial t} \mathbf{A} = \left(\mathbf{u} - \epsilon \frac{\mathbf{J}}{\rho} \right) \times \mathbf{B} - \mu_o \mathbf{J} + \nabla F$$

FLR
εΠ
Hall

$$\Pi_{xx} = 0$$

$$\Pi_{yy} = -\Pi_{zz} = -\frac{P}{2} \left[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right]$$

$$\Pi_{yz} = \Pi_{zy} = \frac{P}{2} \left[\frac{\partial u_y}{\partial y} - \frac{\partial u_z}{\partial z} \right]$$

$$\Pi_{zx} = \Pi_{xz} = P \left[\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right]$$

$$\Pi_{xy} = \Pi_{yx} = -P \left[\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right]$$

CGL (eg, Yajima, 1966)

$$P = \rho^\gamma / (\gamma M_{s0}^2) \text{ (isotropic : } P_{\perp} = P_{\parallel})$$

$$\mathbf{B} = B_0 \hat{x} + \nabla \times \mathbf{A}$$

$$\epsilon = \omega_A / \Omega_i = \omega(k_{\min}) / \Omega_i$$

$$\nu \rightarrow \nu(k) = \nu_0 [1 + (k/k_{eq})^2]$$

$$\mu \rightarrow \mu(k) = \mu_0 [1 + (k/k_{eq})^2]$$

$$F \rightarrow F_{\mathbf{k}} = \frac{i\mathbf{k} \cdot [\mathbf{u} \times \mathbf{B}]_{\mathbf{k}}}{k^2}$$

Wave-wave interactions addressed accurately; wave-particle interactions are absent.

Hall-FLR MHD System

$$\frac{\partial}{\partial t} \mathbf{u} = \mathbf{M}_{\text{Lin}} + \mathbf{M}_{\text{NL}} + \mathbf{M}_{\text{diss}}$$

$$\frac{\partial}{\partial t} \mathbf{B} = \mathbf{N}_{\text{Lin}} + \mathbf{N}_{\text{NL}} + \mathbf{N}_{\text{diss}}$$

Hall-FLR MHD System


$$\frac{\partial}{\partial t} \mathbf{u} = \mathbf{M}_{\text{Lin}} + \mathbf{M}_{\text{NL}} + \mathbf{M}_{\text{diss}} \quad R_M = \frac{\langle |\mathbf{M}_{\text{NL}}| \rangle}{\langle |\mathbf{M}_{\text{Lin}}| \rangle}$$

$$\frac{\partial}{\partial t} \mathbf{B} = \mathbf{N}_{\text{Lin}} + \mathbf{N}_{\text{NL}} + \mathbf{N}_{\text{diss}} \quad R_N = \frac{\langle |\mathbf{N}_{\text{NL}}| \rangle}{\langle |\mathbf{N}_{\text{Lin}}| \rangle}$$

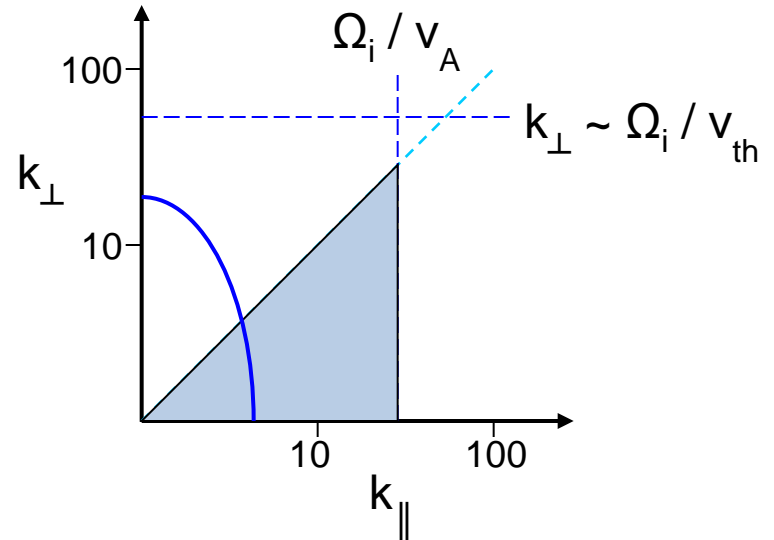
$R_{M,N} \ll 1 \rightarrow$ Linear dynamics

$R_{M,N} \gg 1 \rightarrow$ Nonlinear dynamics

Simulations

<p>Momentum</p> $R_M = \frac{\langle \mathbf{M}_{NL} \rangle}{\langle \mathbf{M}_{Lin} \rangle}$ 	<p>Mag Induction</p> $R_N = \frac{\langle \mathbf{N}_{NL} \rangle}{\langle \mathbf{N}_{Lin} \rangle}$
<p>Momentum</p> $\cos \theta_M = \frac{\mathbf{M}_{Lin} \cdot \mathbf{M}_{NL}}{ \mathbf{M}_{Lin} \mathbf{M}_{NL} }$	<p>Mag Induction</p> $\cos \theta_N = \frac{\mathbf{N}_{Lin} \cdot \mathbf{N}_{NL}}{ \mathbf{N}_{Lin} \mathbf{N}_{NL} }$

Simulation Details



- 2 1/2-D
- 256x256 resolution ($k_{\max} = 128$)
- Dissipation scale: $k \sim 50$
- Hall scale: $k_{\varepsilon} = \Omega_i / V_A = 20$
- FLR scale: $k_L = \Omega_i / v_{\text{th}} = \Omega_i / (\beta^{1/2} V_A) = 10 \dots 20 \dots 40$

Time-Lapse Simulations: VS+Slab



hal51r.avi



hlfr50r.avi

NL/Lin analysis:

$R_M(k_x, k_y)$ & $R_N(k_x, k_y)$

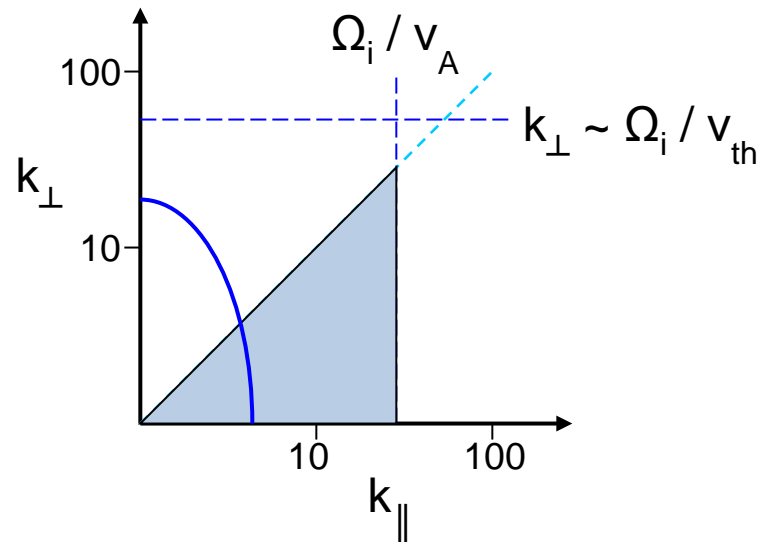
$R_M, R_N \ll 1 \rightarrow$ Linear

$R_M, R_N \geq 1 \rightarrow$ NL

VS+Slab: $\beta = 1/4$

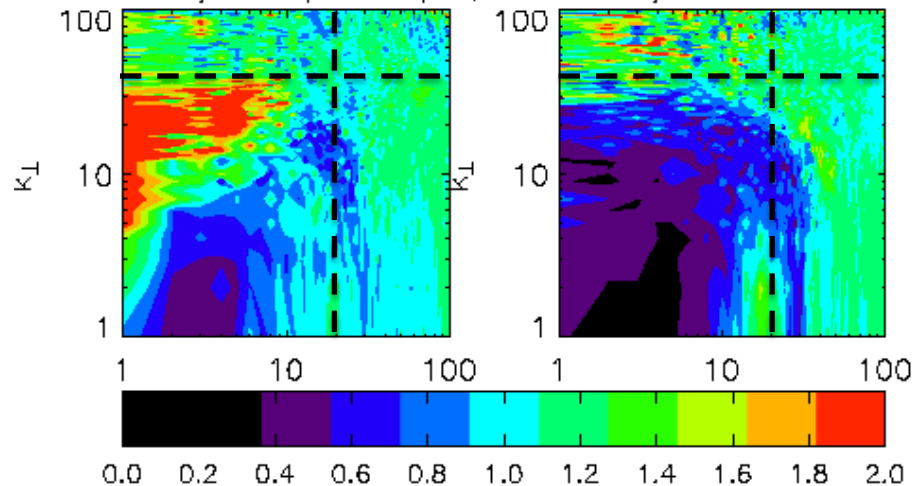
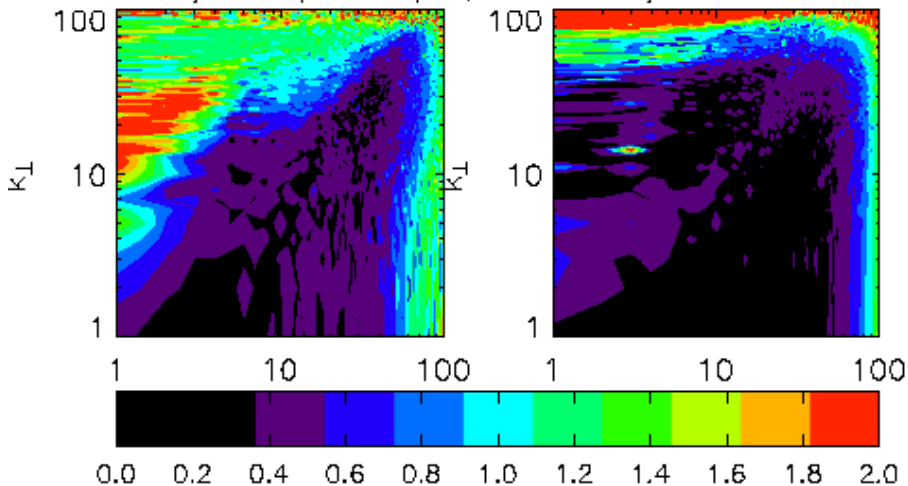
Standard MHD

Hall-FLR MHD



MOM NL/Lin: $\beta = 1/4$; IND NL/Lin: VS+Slab

MOM NL/Lin: $\beta = 1/4$; IND NL/Lin: VS+Slab



NL/Lin analysis:

$$R_M(k_x, k_y) \text{ \& \ } R_N(k_x, k_y)$$

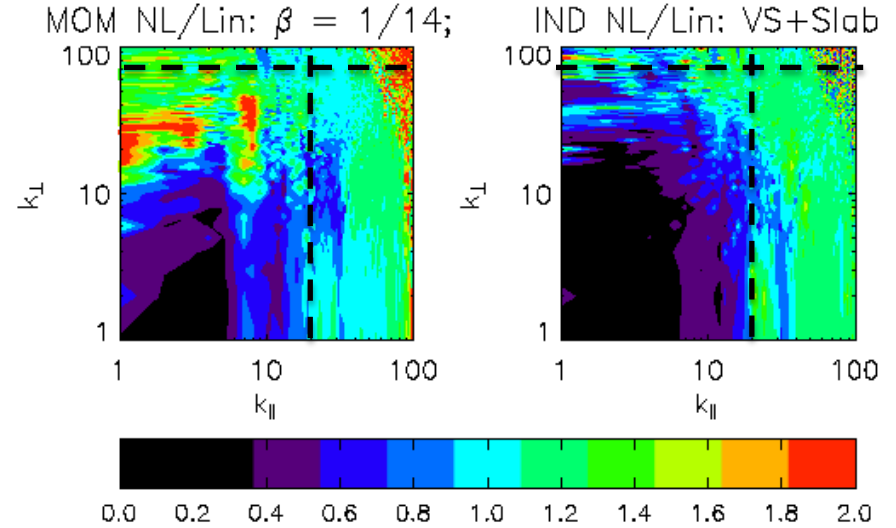
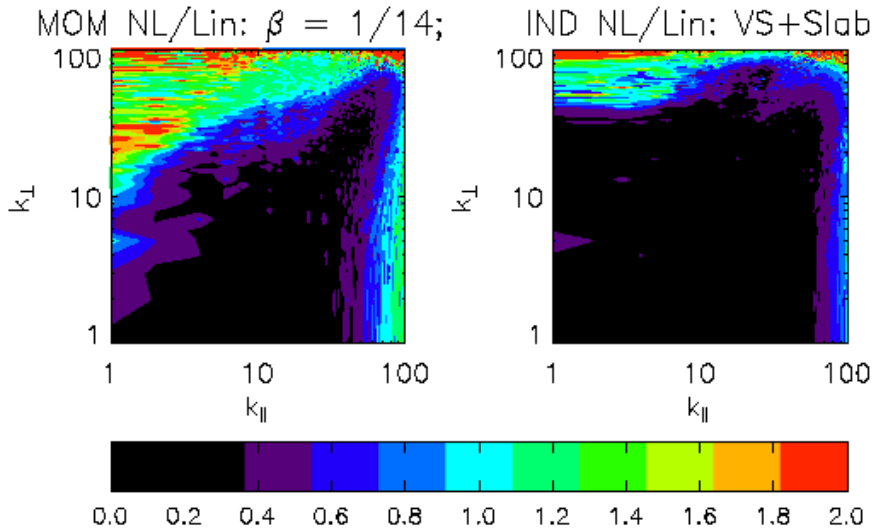
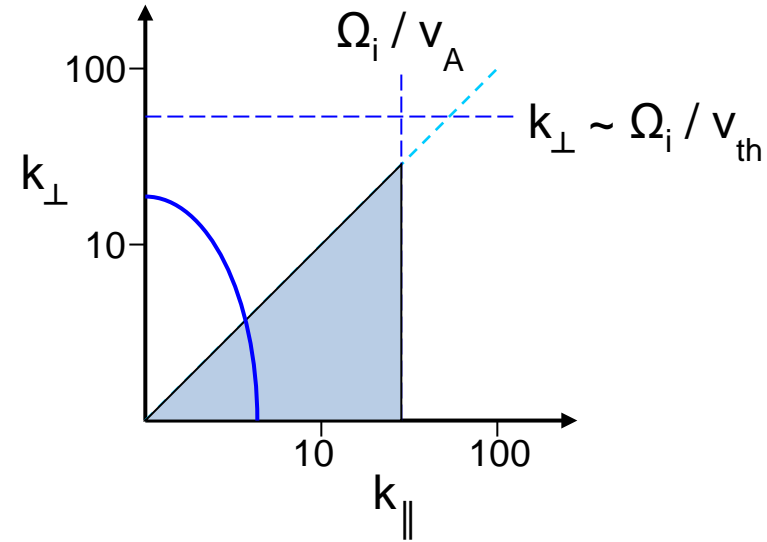
$R_M, R_N \ll 1 \rightarrow$ Linear

$R_M, R_N \geq 1 \rightarrow$ NL

VS+Slab: $\beta = 1/14$

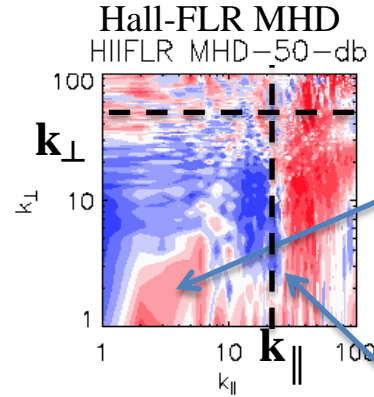
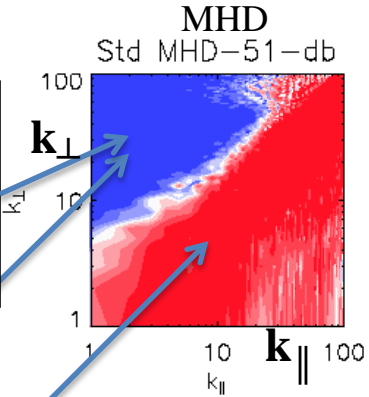
Standard MHD

Hall-FLR MHD

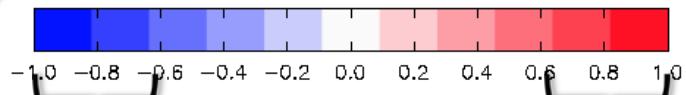


Density-Magnetic Field Correlations

Negative ρ -B correlation associated with NL-dominated k-space regime (Dynamical PB structures)

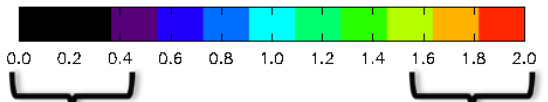
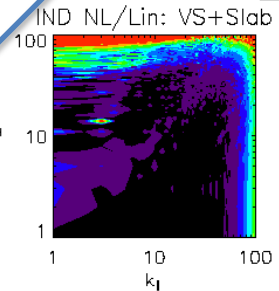
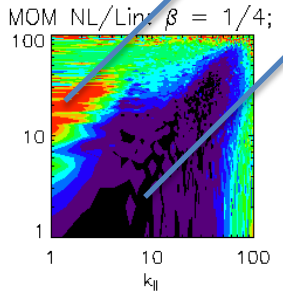


Positive ρ -B correlation associated with Linear-dominated k-space regime (Magnetosonic waves?)



Anti-correlation

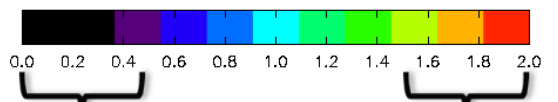
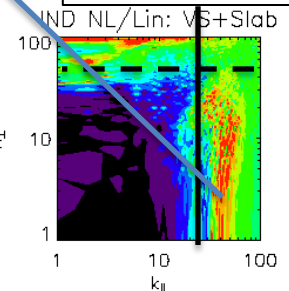
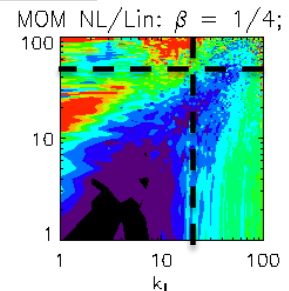
Correlation



Strongly Linear

Strongly NonLinear

Momentum & Mag Ind Equations: NL/Lin Ratio



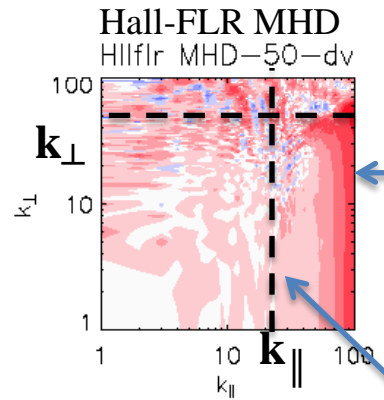
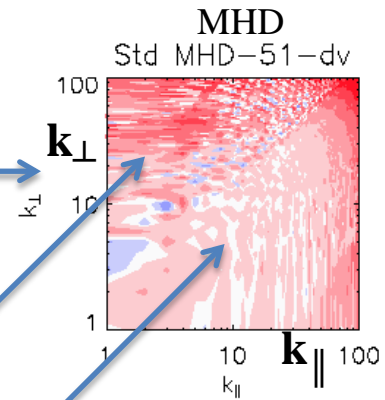
Strongly Linear

Strongly NonLinear

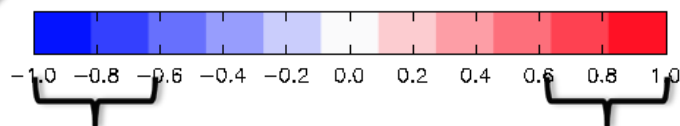
Hall resonance

Density-Longitudinal Velocity Correlations

No significant ρ - u_L Correlation in either Linear or NL regimes

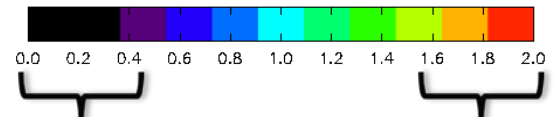
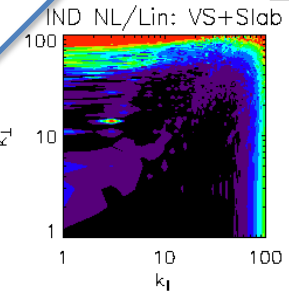
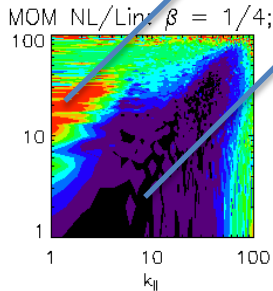


No significant ρ - u_L Correlation in either Linear or NL regimes



Anti-correlation

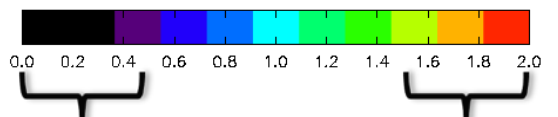
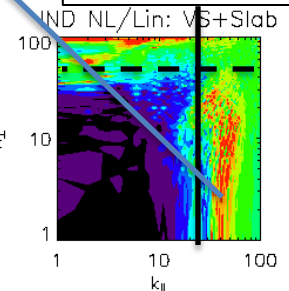
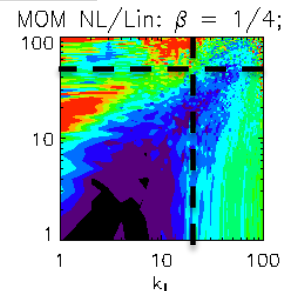
Correlation



Strongly Linear

Strongly NonLinear

Momentum & Mag Ind Equations: NL/Lin Ratio



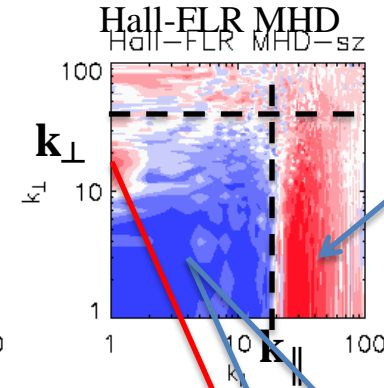
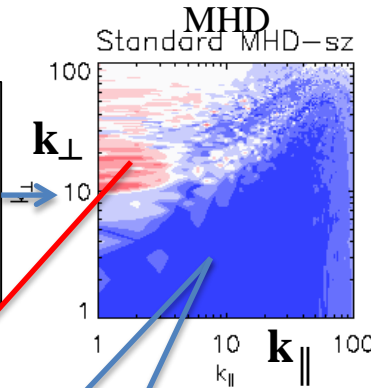
Strongly Linear

Strongly NonLinear

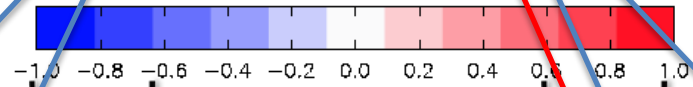
Hall resonance

Velocity-Magnetic Field (Cross-Helicity) Correlations – Early Times

Degradation of negative Hc (minority species effect) – Introduction of z+ along k_{\perp} in primarily z- turbulence.

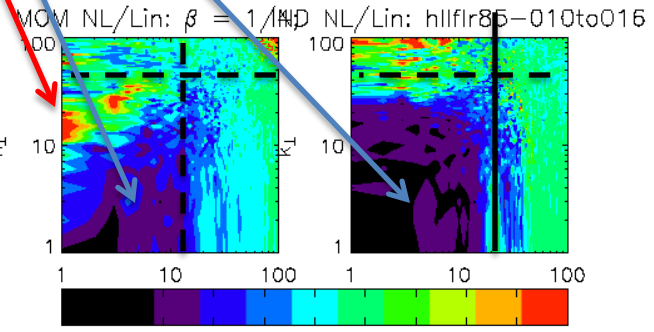
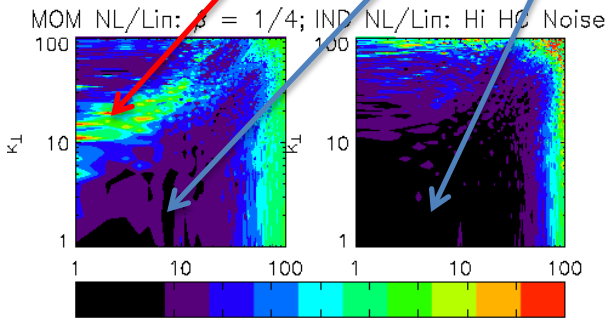


Positive v-B correlation associated with Non-Linear Accelerations (not the standard minority species)



Anti-correlation

Correlation



Hall resonance

Momentum & Mag Ind Equations: NL/Lin Ratio

Strongly Linear

Strongly NonLinear

Strongly Linear

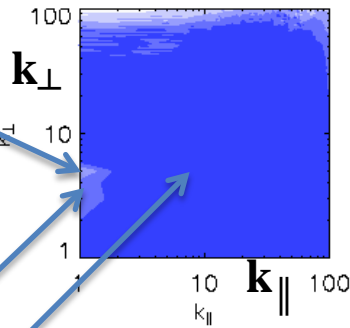
Strongly NonLinear

Velocity-Magnetic Field (Cross-Helicity) Correlations

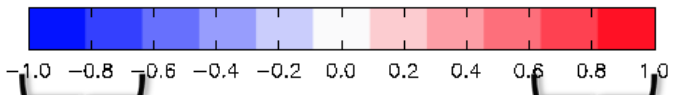
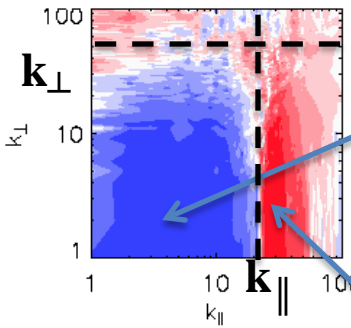
Degradation of negative H_c (minority species effect) – Introduction of $z+$ along k_{\perp} in primarily $z-$ turbulence.

Positive v - B correlation associated with Non-Linear Accelerations (not the standard minority species)

MHD
Std MHD-51-sz



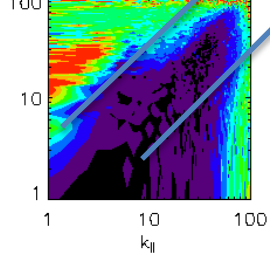
Hall-FLR MHD
HIIFLR MHD-50-sz



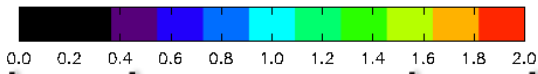
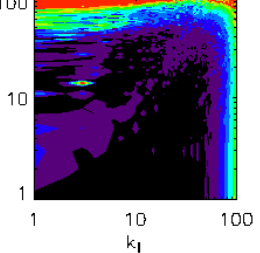
Anti-correlation

Correlation

MOM NL/Lin: $\beta = 1/4$;



IND NL/Lin: VS+Slab

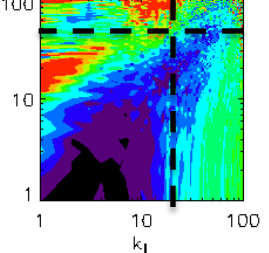


Strongly Linear

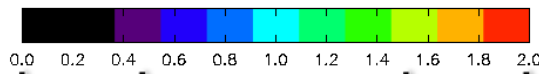
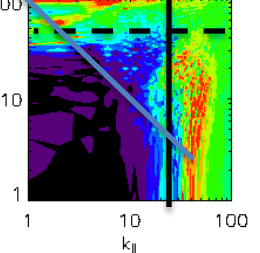
Strongly NonLinear

Momentum & Mag Ind Equations: NL/Lin Ratio

MOM NL/Lin: $\beta = 1/4$;



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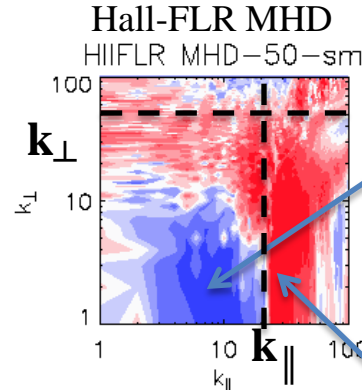
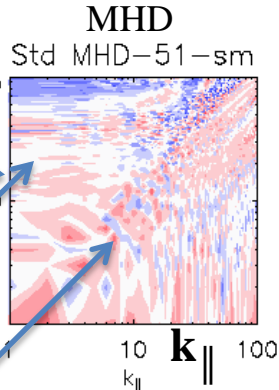
Strongly Linear

Strongly NonLinear

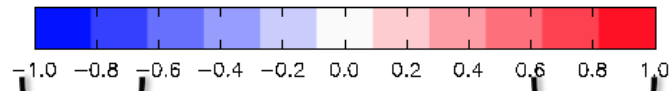
Hall resonance

Vector Potential-Magnetic Field (Magnetic Helicity) Correlations

No evidence of strong σ_m in Standard MHD simulations in either Linear or NL regimes

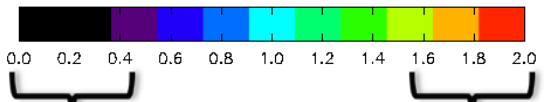
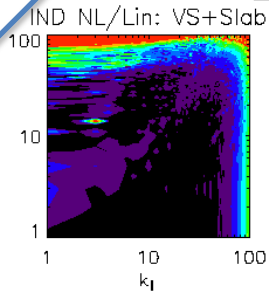
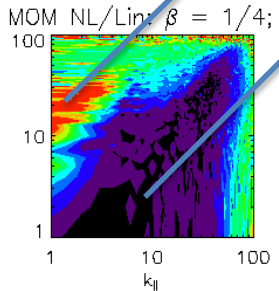


Strong (negative/positive) A-B σ_m correlation in both Linear and NL regimes.



Anti-correlation

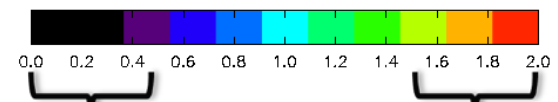
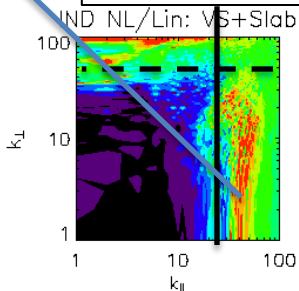
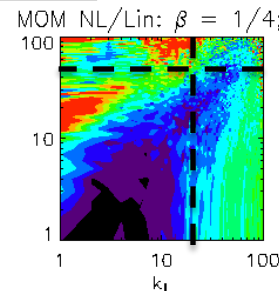
Correlation



Strongly Linear

Strongly NonLinear

Momentum & Mag Ind Equations: NL/Lin Ratio

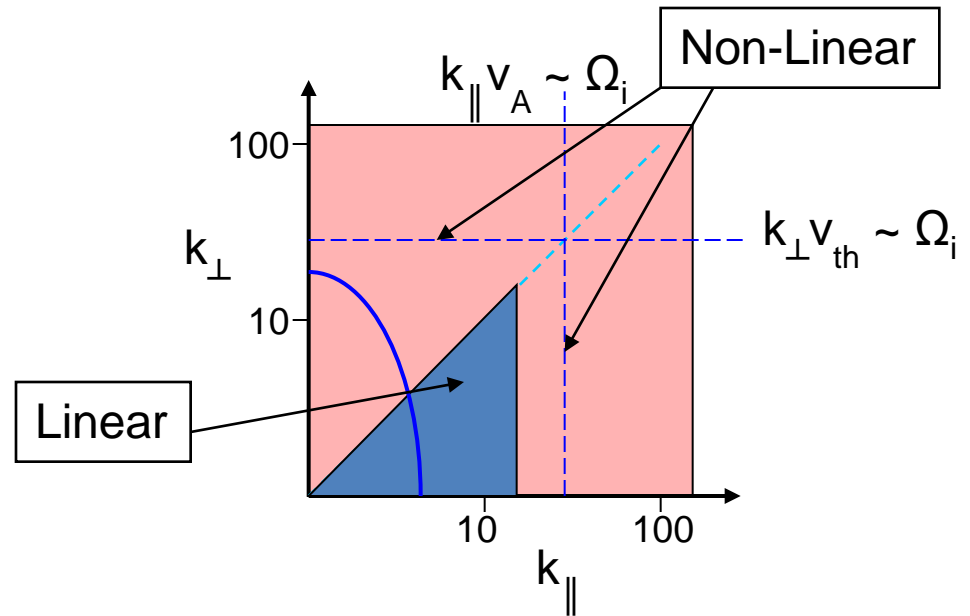


Strongly Linear

Strongly NonLinear

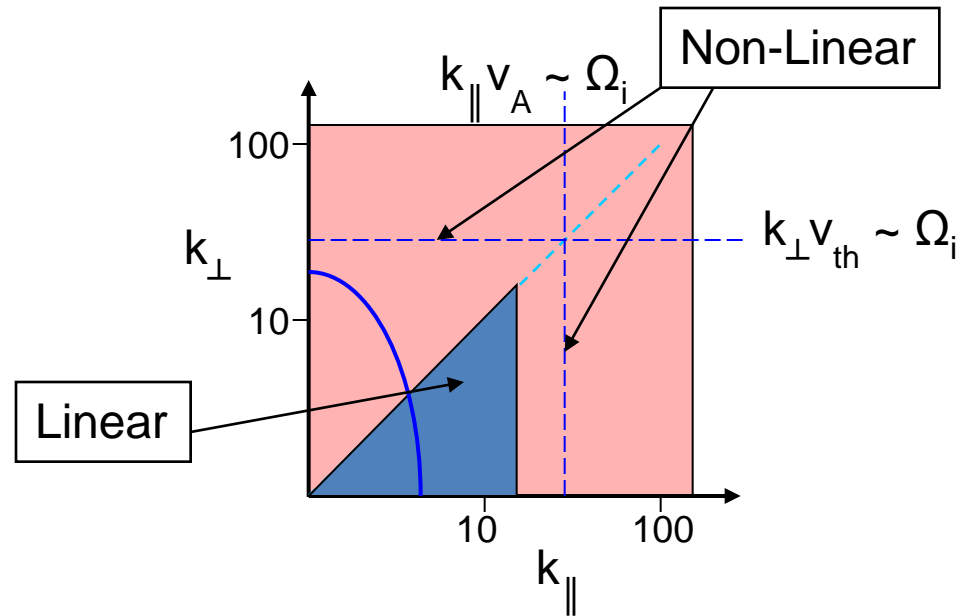
Hall resonance

Conclusions



- Non-linear forces dominate across a broad region of k-space – especially at angles $\theta > 45^\circ$ to \mathbf{B}_0 .
- Linear forces persist in a limited region of k-space within angles $\theta < 45^\circ$ to \mathbf{B}_0 .
- Dynamics at Hall-FLR scales appear governed by strongly nonlinear influences.
- Hence, linear treatments (e.g., KAW) may not be appropriate adjacent the dissipation scales in the Solar Wind.

Conclusions (cont'd)



- Correlations (ρ -B , σ_z , σ_m) adjacent the Hall-FLR / dissipation scales may be governed by strongly nonlinear influences; linear theory may not be applicable.
- Disclaimer:
 - Fluid treatment may not apply at ion-cyclotron and smaller scales.