Compressible MHD Turbulence

Theoretical and astrophysical interests

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Basic Theoretical Assumptions

- Compressible MHD fluid with isothermal closure ($P = C_s^2 \rho$)
- Statistical Homogeneity
- Existence of a stationary state corresponding to the total energy (kinetic + magnetic + compressible) conservation
- Existence of an inertial zone (independent of the external forcing and the dissipation)

Exact relation for isothermal CMHD turbulence ($S.Banerjee$ & $S.Galtier$ PRE, 2013)

In the inertial zone & without assuming isotropy

$$-2\varepsilon = \frac{1}{2}\nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{1}{2}\delta(\rho\mathbf{z}^-)\cdot\delta\mathbf{z}^- + \delta\rho\delta e\right]\delta\mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho\mathbf{z}^+)\cdot\delta\mathbf{z}^+ + \delta\rho\delta e\right]\delta\mathbf{z}^- + \overline{\delta}(e + \frac{v_A^2}{2})\delta(\rho\mathbf{z}^- + \rho\mathbf{z}^+)\right\rangle$$

$$-\frac{1}{8}\left\langle \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^+e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^+e) + \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^-e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^-e)\right\rangle$$

$$+\left\langle(\nabla\cdot\mathbf{v})\left[R'_E - E' - \frac{\overline{\delta}\rho}{2}(\mathbf{v_A}'\cdot\mathbf{v_A}) - \frac{P'}{2} + \frac{P'_M}{2}\right]\right\rangle + \left\langle(\nabla'\cdot\mathbf{v}')\left[R_E - E - \frac{\overline{\delta}\rho}{2}(\mathbf{v_A}\cdot\mathbf{v_A}') - \frac{P}{2} + \frac{P_M}{2}\right]\right\rangle$$

$$+\left\langle(\nabla\cdot\mathbf{v_A})\left[R_H - R'_H + H' - \overline{\delta}\rho(\mathbf{v}'\cdot\mathbf{v_A})\right]\right\rangle + \left\langle(\nabla'\cdot\mathbf{v_A}')\left[R'_H - R_H + H - \overline{\delta}\rho(\mathbf{v}\cdot\mathbf{v_A}')\right]\right\rangle$$

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Reduced form in the presence of a strong $\mathbf{B_0}$

$$-2\varepsilon \simeq \frac{B_0^2}{2}\nabla_{r_\perp}\cdot\left\langle\delta\left(\frac{1}{\sqrt{\rho}}\right)\delta(\sqrt{\rho})\delta\mathbf{v}_\perp\right\rangle - \frac{B_0^2}{4}\left\langle(\nabla_\perp\cdot\mathbf{v}_\perp)\left(1+\sqrt{\frac{\rho}{\rho'}}\right) + (\nabla_\perp'\cdot\mathbf{v}_\perp')\left(1+\sqrt{\frac{\rho'}{\rho}}\right)\right\rangle$$

Important Inferences

- 2 types of flux terms and 2 types of source terms

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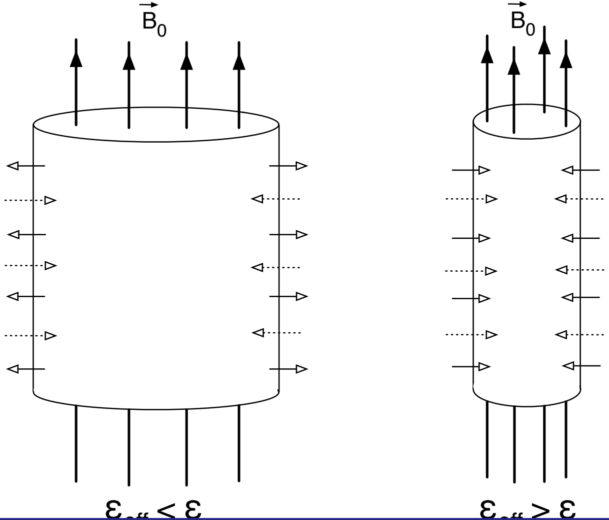
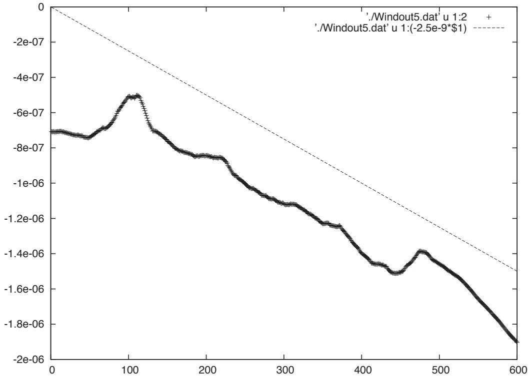
- For weak compressibility, 3rd order moment scaling can be expected taking $(\rho {z^+}^2 z^- + \rho {z^-}^2 z^+)^{1/3}$ as the scaling variable

Phenomenology for Reduced Case



Scaling in solar wind data

Scaling with 1 day of WIND data with 1 minute plasma resolution



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- How to estimate the $\mathbf{B_0}$ for solar wind ?
- Can we have a phenomenological view for the general case of compressible MHD turbulence ?