Energy Loss Under Guiding Center Motion

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<u>Breakout Session D</u>:
Pickup Ions and Building the Energetic Particle Reservoir

Basic theory-relativistic energy loss rate [*Northrop*, 1963]

$dT/dt = ZeV_{||}E_{||}+(mv_\perp^2/2)(\partial lnB/\partial t+\mathbf{u}_E\cdot\nabla lnB)+mv_{||}^2\ \mathbf{u}_E\cdot(\mathbf{b}\cdot\nabla\mathbf{b})$

$(mv_\perp^2/2)(\partial lnB/\partial t)$ is betatron term

$\mathbf{u}_E=\mathbf{E}\times\mathbf{B}/B^2$ and $\mathbf{b}\cdot\nabla\mathbf{b} = \boldsymbol{\kappa}$ so remaining terms are

$\mathbf{u}_E\cdot(mv_\perp^2/2\nabla lnB+mv_{||}^2\boldsymbol{\kappa}) = (1/B)\mathbf{E}\cdot[\mathbf{b}\times(mv_\perp^2/2\nabla lnB+mv_{||}^2\boldsymbol{\kappa})]$

Relativistic gradient and curvature drift velocities are

$\mathbf{u}_G = (M/\gamma ZeB)\mathbf{b}\times\nabla B$ $\mathbf{u}_C = (p_{||}^2/\gamma Zem_0B)\mathbf{b}\times\boldsymbol{\kappa}$

relativisitic magnetic moment $M=p_\perp^2/2m_0$ p. 29

Terms equal $Ze\mathbf{E}\cdot(\mathbf{u}_G+\mathbf{u}_C)$: *work done on drifting particle by the electric field.*

Momentum fractional time rate of change

$dT = v\,dp$ so $dT/dt = v\,dp/dt$

$$dp/dt = Ze(V_{||}/v)E_{||} + p(1-\mu^2)/2(\partial ln B/\partial t + \mathbf{u}_E \cdot \nabla ln B) + p\mu^2\, \mathbf{u}_E \cdot (\mathbf{b} \cdot \nabla \mathbf{b})$$

Neglecting the $E_{||}$ term and setting $\mathbf{u}_E = \mathbf{V}_\perp$ and $\mathbf{b} \cdot \nabla \mathbf{b} = \boldsymbol{\kappa}$, we have the result [*Roelof*, 1999; 2000]

$$d\,ln\,p/dt = (1-\mu^2)/2(\partial ln B/\partial t + \mathbf{V}_\perp \cdot \nabla ln B) + \mu^2\, \mathbf{V}_\perp \cdot \boldsymbol{\kappa}$$

Remark: The fractional rate is ***independent*** of particle mass or charge! It depends only on the particle pitch-angle, the component of the transverse plasma velocity and the local spatial-temporal variations in the magnetic field.

Parker field

$B_r = B_0(r/r_0)^2$ $B_\Theta = 0$ $B_\Phi = -B_r(\Omega r \sin\Theta/V) = -B_r \tan\psi$

General identities

$\nabla_\perp \ln B = (1/B^2)\mathbf{B}\times(\nabla\times\mathbf{B}) + \boldsymbol{\kappa}$ $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$

$\mu_0\mathbf{J} = -2(\Omega/V)B_0(r_0/r)^2\cos\Theta\ \mathbf{u}_r$

$\mu_0\mathbf{J}\times\mathbf{B} = -(\Omega B_0/V)^2(r_0^{\,4}/r^3)\sin 2\Theta\ \mathbf{u}_\Theta$

thus $\mathbf{V}_\perp\cdot\mathbf{J}\times\mathbf{B}=0$ and so $\mathbf{V}_\perp\cdot\nabla_\perp \ln B = \mathbf{V}_\perp\cdot\boldsymbol{\kappa}$ for the Parker field

$\mathbf{V}_\perp = (\boldsymbol{I}\text{-}\mathbf{bb})\cdot\mathbf{V} = V(\mathbf{u}_r \sin^2\psi + \mathbf{u}_\phi \sin\psi\cos\psi)$

$B^2 = B_0^2 (r_0/r)^4 (1+\tan^2\psi)$ $\partial \tan\psi / \partial \ln r = \tan\psi$

$\mathbf{V}_\perp \cdot \nabla \ln B = -(V/r)(1-\cos^4\psi) = \boldsymbol{\kappa}\cdot\mathbf{V}_\perp$

$d\ln p/dt = (1-\mu^2)/2(\partial \ln B/\partial t + \mathbf{V}_\perp \cdot \nabla \ln B) + \mu^2\, \mathbf{V}_\perp \cdot \boldsymbol{\kappa}$

$= (1-\mu^2)/2\ \partial \ln B/\partial t - (1+\mu^2)(V/2r)(1-\cos^4\psi)$

[*Roelof*, Proc. 7th Int. Astrophys. Conf., *AIP*, **1039**, 174-183, 2008]

Limit: $\tan\psi >> 1$ or $r\sin\Theta >> V/\Omega \sim 1$AU
Also $\partial \ln B/\partial t = 0$, and isotropic p.a.d.

$\langle d\ln p/dt \rangle \rightarrow -2V/3r$ ***"adiabatic cooling"***

General expression for fractional momentum rate

$d\ln p/dt = (1-\mu^2)/2\ \partial \ln B/\partial t-(1+\mu^2)(V/2r)(1-\cos^4\psi)$

$\partial \ln B/\partial t=0 \qquad d\ln p/dt = -(1+\mu^2)(V/2r)(1-\cos^4\psi)$

$= -(1+\mu^2)(V/2r)[\sin^2\psi+(1/4)\sin^2 2\psi]$

Limit: $\tan\psi<<1$ or $r\sin\Theta<<V/\Omega\sim 1AU$

$d\ln p/dt\rightarrow -(V/2r)(1+\mu^2)2(\Omega r\sin\Theta/V)^2$

$= -(1+\mu^2)(\Omega\sin\Theta)^2(r/V) \rightarrow 0$ as $r \rightarrow 0$

No "adiabatic deceleration" near the Sun!

Conclusions for guiding center energy loss in a Parker magnetic field (applicable to "scatter-free" propagation)

Outside Earth's orbit: $r\sin\Theta > V/\Omega \sim 1\text{AU}$

$\langle d\,ln\,p/dt \rangle \rightarrow -2V/3r$ ***"adiabatic cooling"***

Inside Earth's orbit: $r\sin\Theta < V/\Omega \sim 1\text{AU}$

$d\,ln\,p/dt = (1-\mu^2)/2\ \partial\,ln B/\partial t - (1+\mu^2)(V/2r)(1-\cos^4\psi)$

$\rightarrow\ -(1+\mu^2)(\Omega\sin\Theta)^2(r/V) \rightarrow 0$ as $r \rightarrow 0$

No "adiabatic deceleration" near the Sun!—for *any* nearly "scatter-free" particle (PUIs or GCRs)