On the Acceleration of the Frequently Observed v^{-5} Spectrum in the Heliosphere

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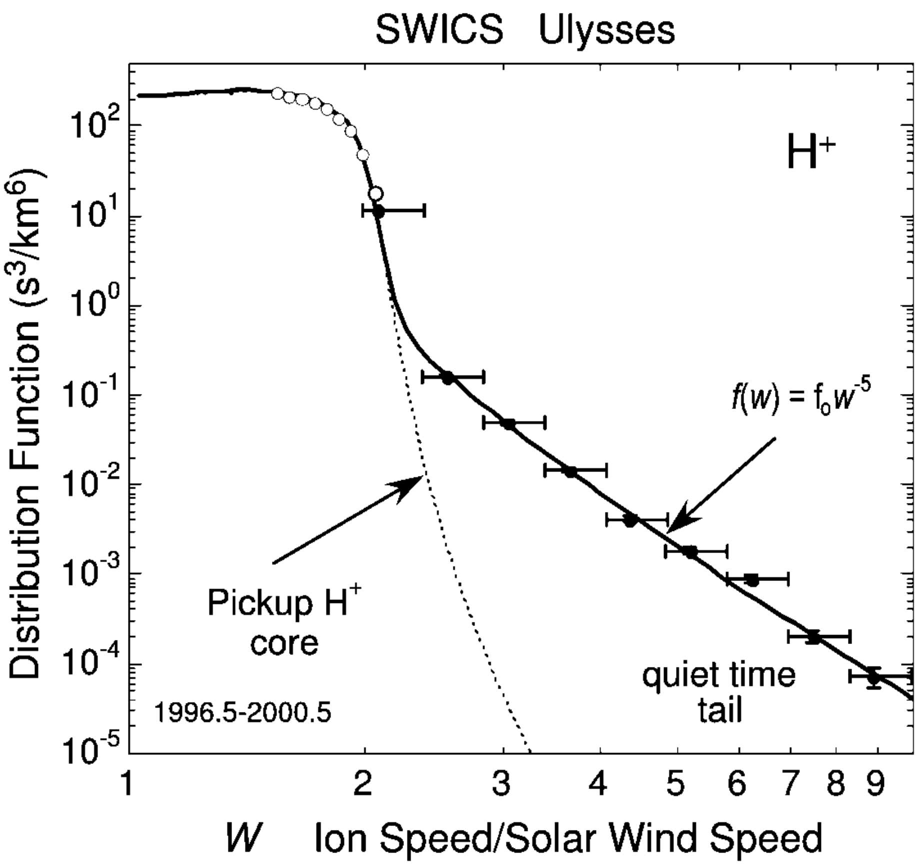
From Gloeckler and Fisk, 2006



Note: 10 times the wind speed = 250 keV in the fast wind

The main question is:
what produces the superthermal particles?

Background

- Fisk and Gloeckler (F & G)have published an extensive series of papers on observed superthermal tails and the nature of their velocity spectrum – very frequently $f(v) \propto v^{-5}$.

- A paper of theirs appeared in 2008, which presented a theory of the $f(v) \propto v^{-5}$ spectrum, based ondiffusive-compression acceleration.

- Jokipii & Lee (J&L) published paper on compression acceleration which included comments on the Fisk & Lee 2008 paper. (Ap. J. April 10).

The acceleration of a charged particle in a collisionless MHD fluid is given by:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} \mathbf{w} \cdot \mathbf{E}(\mathbf{r},\mathbf{t})dt$$
$$= \int_{t_0}^{t_0+\Delta t} \mathbf{w} \cdot \frac{\mathbf{U}(\mathbf{r},\mathbf{t}) \times \mathbf{B}(\mathbf{r},\mathbf{t})}{c} dt$$

One may use the latter relation to express the energy change in terms of **B** and **U**. Since the position is needed we must also study spatial transport as well.

The Parker Equation can be used

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i}\left[\kappa_{ij}^{(S)} \frac{\partial f}{\partial x_j}\right]$$ ⇒ **Diffusion**

$$-\mathbf{U} \cdot \nabla f$$ ⇒ **Convection w. plasma**

$$-\mathbf{V_d} \cdot \nabla f$$ ⇒ **Grad & Curvature Drift**

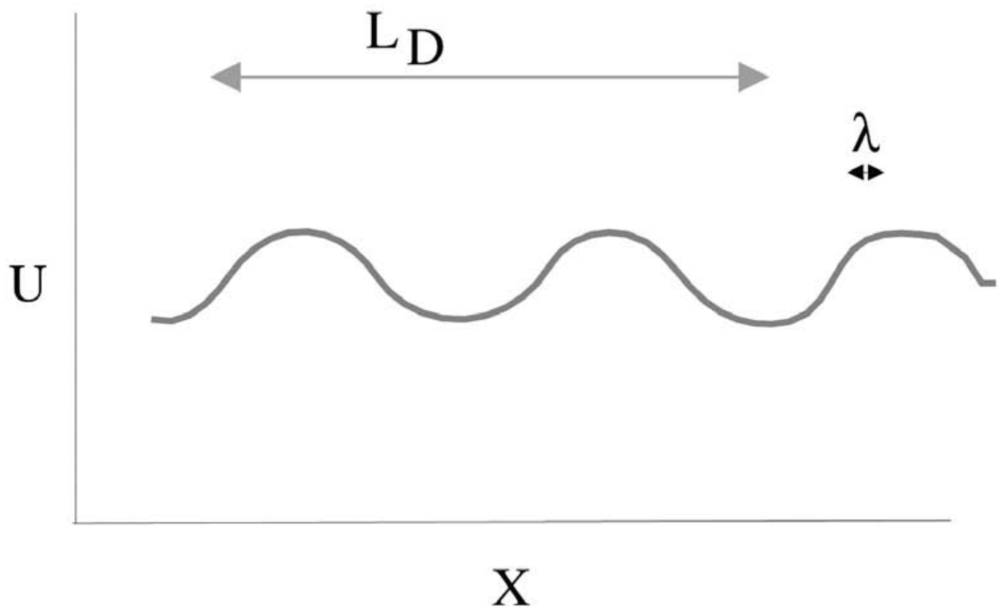
$$+\frac{1}{3}\nabla \cdot \mathbf{U}\left[\frac{\partial f}{\partial \ell np}\right]$$ ⇒ **Energy change**

$$+Q$$ ⇒ **Source**

Where the drift velocity is: $\mathbf{V_d} = \frac{pcw}{3q} \nabla \times \left[\frac{\mathbf{B}}{B^2}\right]$

This equation applies if there is enough scattering to produce near-isotropy.

Compression Acceleration (suggested ~ 30 years ago)



This is a cartoon representation of compression acceleration for slow fluctuations in the fluid velocity.

The diffusion length $L_D = \kappa/\delta U$ must be larger than the spatial scale of the fluid fluctuations. Hence the particle samples many random velocity fluctuations and diffuses in velocity.

But, at the same time, the scattering mean free path λ must be small compared with the velocity fluctuation scale so that diffusion applies.

Particle Conservation is fundamental.

In 3-D physical space, x,y,z , we have

But if it is spherically symmetric with a radial velocity U_r

+ Q – L (source - loss)

Hence, in *velocity space*, with *isotropic velocities*, the velocity magnitude rate of change becomes the *acceleration* ***a***

+ Q – L (source – loss)

The Parker equation can be rewritten in the manifestly conservation-law form

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x_i}\left[-(\kappa_{ij}^{(S)} + \kappa_{ij}^{(A)})\frac{\partial f}{\partial x_j} + U_i f\right]$$ ⇒ **- div (flux in space)**

$$-\frac{1}{v^2}\left[\frac{\partial}{\partial v}v^2\frac{-\nabla\cdot\mathbf{U}}{3}vf\right]$$ ⇒ **- div (flux in velocity)**

$$+Q - L$$ ⇒ **Source - Loss**

This equation requires only enough scattering to maintain near- isotropy. It is the starting point of all existing discussions of compression acceleration.

J&L Ap. J. 2010

- The analysis is to lowest order in δu (in an approximation called quasilinear theory).
-
- In this approximation, *the resulting equation is diffusion in velocity*.

- We include the possibility of drifts and gradients in κ, but they do not appear to this order.

Diffusive compression acceleration for the simple case where $U = \delta U$ has zero mean and $f = \langle f \rangle + \delta f = f_0 + \delta f$, where f_0 is constant (*homogeneous in space*), can be derived in one spatial dimension (x) as follows:

Take the average of Parker's equation to obtain

Subtracting this from Parker's equation yields (discarding second-order terms) :

Note: hereinafter κ is the spatial diffusion coefficient.

This can be solved for δf in terms of the diffusion solution for diffusion from a point source at x':

to obtain (going now back to 3 spatial dimensions):

Substitution of this into the original equation for f_0 yields

Now we must make one *critical assumption*, which is not often mentioned. The solution for δf solves only the linearized equation. It is only valid for a finite time depending on the magnitude of δU. Call this time t_1

Our solution has 'history' terms. These will go to zero if t_1 is much greater than the *coherence* time of δU, but small enough that the approximation is valid. Thus this is a multiple-time-scale problem.

The time for the validity of the approximation must be greater than the coherence scale. Then we obtain a Markov process (velocity diffusion) - independent of the history.

We finally obtain the transport equation:

This is diffusion in velocity. Note the $1/v^2$

One example: the solution for $D v^2 = D' = \text{const}$, for a source at v_0 is

In contrast, Fisk and Gloeckler's (2008) Transport Equation:

- Fisk and Gloeckler proposed an equation of the form:

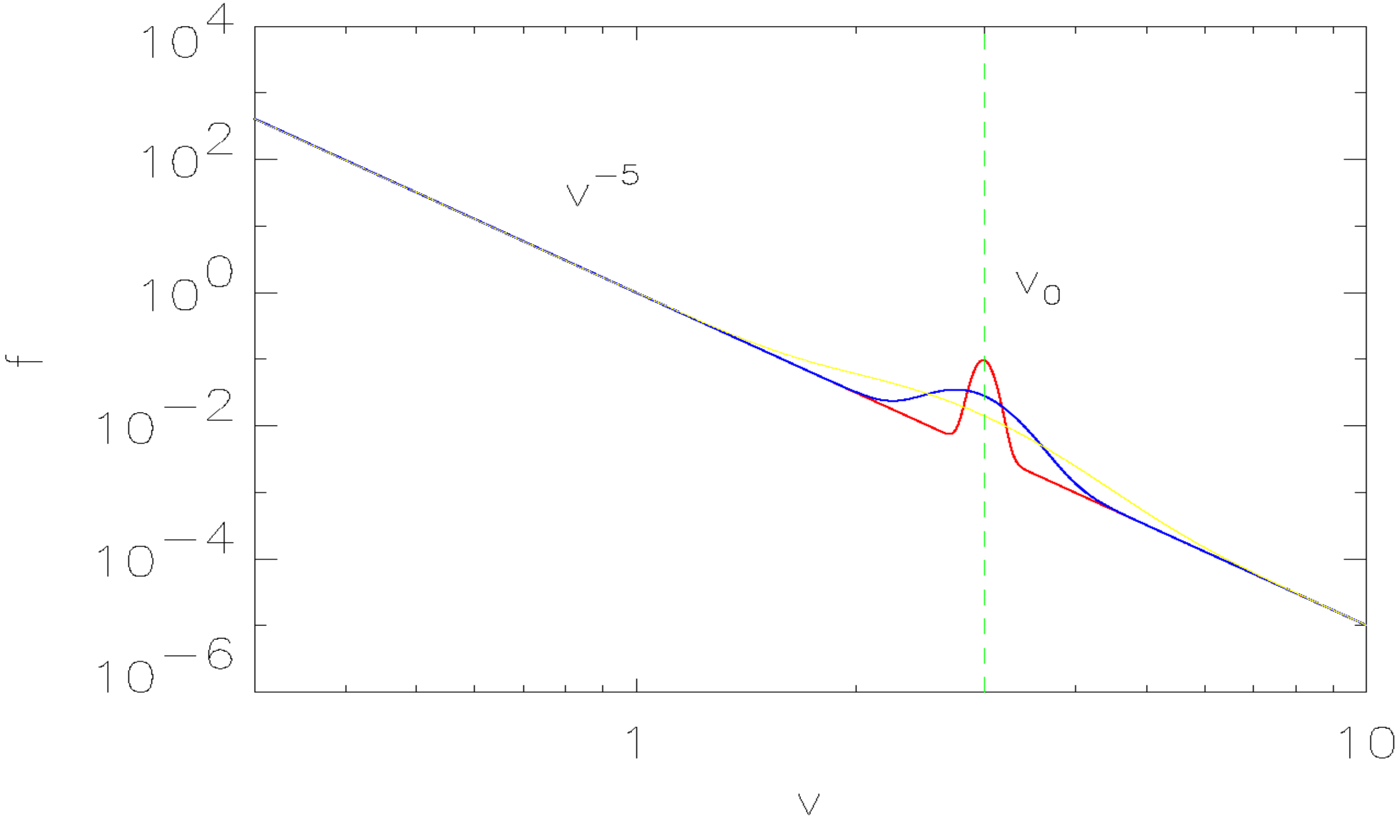
$$\frac{\partial f}{\partial t} = \frac{1}{v^4}\frac{\partial}{\partial v}\left[\frac{< \delta u^2 >}{9\kappa} v \frac{\partial}{\partial v}(v^5 f)\right]$$

- Clearly, this has a possible steady state f $\propto$ v^{-5}. f has an additional spatial dependence which is not shown. This does not affect the argument.
- It can be shown that this equation does not conserve particles: define

$$N(t) = \int_0^\infty 4\pi v^2 f(v) dv$$

Then dn/dt is not zero in general. This is because their equation is not in conservation-law form.

As one example, consider the initial, steady spectrum Av^{-5} and then superpose a small bump at velocity v_0. For a spatial diffusion coefficient κ which is independent of v, we find the time-dependent solution:



This solution can be integrated to find the change in N with time:

The number of particles for a small fluctuation, starting at *any* velocity v_0 grows *exponentially*. This can be shown numerically to be also true for other forms of κ.

Since v_0 can be any velocity, we conclude that the equation proposed by Fisk and Gloeckler in 2008 does not conserve particles at *any* velocity. This is *not* a boundary issue.

General Constraints on Acceleration

- There are many recent issues and new proposals concerning the acceleration of fast charged particles in the Sun and heliosphere.
- I will discuss fundamental constraints – spatial, temporal.
- Statistical Acceleration (e.g., G&F)
- Reconnection (Lazarian, Drake).

From F&G -- latest:

We divide f into its mean, or ensemble-averaged value, f_0, and the deviation from the mean, δf, or $f = f_0 + \delta f$, and then group the terms of equation (4) as

$$\frac{\partial f_o}{\partial t} - \frac{(\nabla \bullet \delta \mathbf{u})}{3v^4}\frac{\partial}{\partial v}\left(v^5 f_0\right) - \frac{(\nabla \bullet \delta \mathbf{u})}{3v^4}\frac{\partial}{\partial v}\left(v^5 \delta f\right) + \frac{\delta f}{\tau} = -\left[\frac{\partial \delta f}{\partial t} + \delta \mathbf{u} \bullet \nabla \left(f_o + \delta f\right) + \frac{5}{3}(\nabla \bullet \delta \mathbf{u})\left(f_o + \delta f\right)\right]. \quad (5)$$

We make an assumption that the terms on the right side of equation (5) sum to zero, or

$$\frac{\partial \delta f}{\partial t} + \delta \mathbf{u} \bullet \nabla \left(f_o + \delta f\right) + \frac{5}{3}(\nabla \bullet \delta \mathbf{u})\left(f_o + \delta f\right) = 0. \quad (6)$$

This assumption, which is justified in the next section, permits us to perform the acceleration through a series of adiabatic compressions and expansions, and thus the turbulence is not damped.

NOTE: this is the step that is the source of the disagreement.

The energy change of a charged particle in the absence of collisions comes from the electric force. In a collisionless MHD fluid this may be written:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} \mathbf{w} \cdot \mathbf{E}(\mathbf{r},\mathbf{t}) dt$$
$$= -\int_{t_0}^{t_0+\Delta t} \mathbf{w} \cdot \frac{\mathbf{U}(\mathbf{r},\mathbf{t}) \times \mathbf{B}(\mathbf{r},\mathbf{t})}{c} dt$$

One may use the second relation above to express the energy change in terms of **B** and **U**. Since the position is needed we must include spatial transport as well.

Observational Constraints on the Time Taken to Accelerate

- SEP ions and electrons – acceleration time apparently less than minutes. Significant temporal changes occur in seconds, but these may not reflect acceleration times. These are not significant constraints now.

- Heliospheric Particles
  – The only real constraint is the time available for acceleration. At present these are not significant constraints.

- Anomalous Cosmic Rays
  – The observed ACR charge states limit the acceleration time of ACR to less than a few years (e.g., Adams, 1991; Jokipii, 1992; Mewaldt, etal, 1996).

Spatial Constraints

- Larger systems can accelerate to higher energies.

- In quasi-static flows such as shocks and reconnection events, the available electric potential is relevant and can be a serious constraint.

Stochastic vs Deterministic Acceleration

- Stochastic acceleration
  - Example: 2nd-order Fermi
  - Involves a *random walk* or *diffusion* in energys.

- Deterministic acceleration
  - Examples: Diffusive shock acceleration, reconnection.
  - Usually is uni-directional in energy – can involve a directed electric field.

Stochastic Acceleration

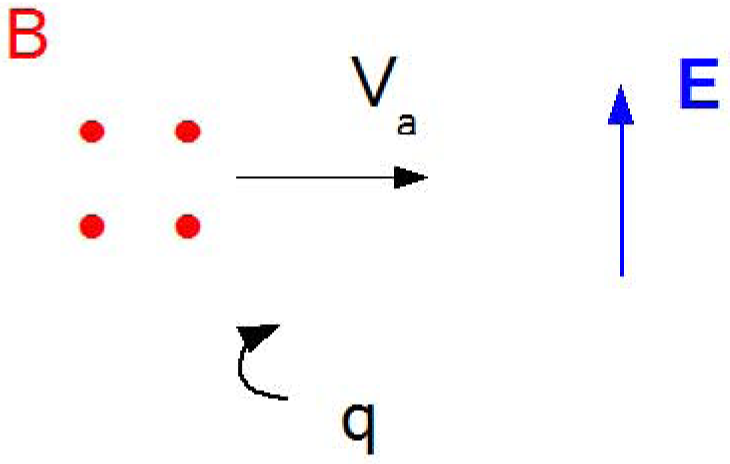
- Has appeared in various forms since Fermi's famous paper on 2nd-order Fermi acceleration by randomly moving magnetic clouds.
- The acceleration time may be approximated, quite generally, as

  where τ_{scat} is the time for magnetic scattering.
- The lowest value of τ_{st} is clearly when $\tau_{scat} = \tau_{gyro}$, the particle cyclotron period.
- This is generally very slow. Applying these considerations to the heliosheath and ACR yields $\tau_{st} \approx 100$ yr, which is much too long.

Stochastic rate of energy change:

In one interaction, depending on the sign of V_a or B, $\Delta T =$ (+/-) $q E r_g = q r_g V_a B/c$. Hence, after n interactions the total mean square energy change is $<(T-T_0)^2> = n \Delta T^2$. where $n < \text{time}/\tau_g$.

The maximum acceleration *rate* is then
$\mathbf{1/\tau_{st,\,max} \approx (V_a^2 /w^2)\, 1/ \tau_g} \approx Va^2 \mathbf{/(w\, \kappa_{B})}$. *This is the same as on the previous slide.*



Finite-Scale Compressions

- Consider compressions and expansions of finite scale -> δu with scale λ. Suppose particles remain a characteristic time τ in each compression or expansion.
- If the diffusion coefficient is κ, then $\tau = \lambda^2/\kappa$
- We then have the change in energy T, $\Delta T \approx (\delta u / \lambda)\, T\, \tau$.
- Then the energy diffusion coefficient, $D_{tt} \approx \langle(\Delta T)^2\rangle/\tau = \langle(\delta u)^2\rangle/\kappa\, T^2$
- Depending on the value of \kappa, we get different energy diffusion rates. Len Fisk pointed out that if κ is the perpendicular diffusion, $\kappa_\perp$, which can be much less than the Bohm limit we can beat my upper limit,.
- But, in this limit, we should also worry about guiding-center drifts. $V_d \approx w\, r_g/\lambda$, where w = particle speed.
- Setting $\tau = \lambda/V_d$ in the above gets back to my original limit which can be written $\langle(\delta u)^2\rangle /(w\, r_g)$.

Deterministic Acceleration: The Role of Electrostatic Potential Energy

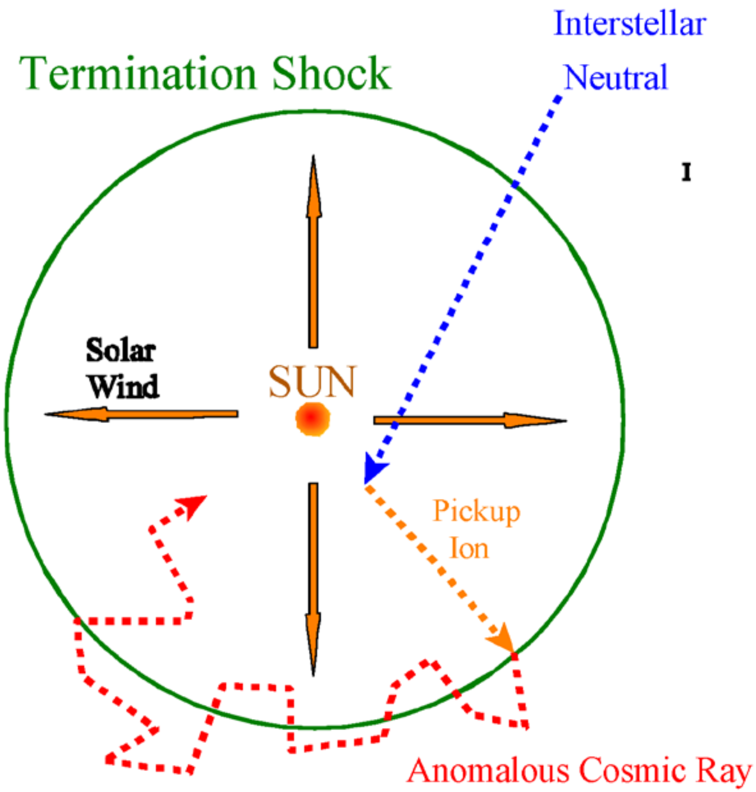
- In an MHD fluid, the electric field $\mathbf{E} = -\mathbf{U} \times \mathbf{B}/c$ is specified by the flow velocity and magnetic field.

- In approximately steady flows such as in quasi-perpendicular shocks and reconnection, the maximum energy is just $T_{max} \approx q \int \mathbf{E} \cdot d\mathbf{l} \approx q \, \Delta \, \phi \approx qUBL/c$.

- Example: using this in the *latitudinal* direction in the heliosphere, integrating from 0 to $\pi/2$, at *any* fixed radius R in the solar wind in the$T_{max} \approx$ 300 Z MeV.

- Applying this to the *heliospheric termination shock* then readily yields the ≈ 200 MeV/charge anomalous cosmic-ray (ACR) energy. *The termination shock can readily give us the ACR.*

ANOMALOUS COSMIC RAYS



The solar-wind termination shock is essentially a perpendicular shock. Hence the energy gain comes from drift in the **-U x B/**c electric field.

The change in electric potential between the pole and the equator is ≈ 300 Z MeV

Hence this this can readily provide the 200 MeV kinetic energy.

Lazarian and Opher (2009) proposed turbulent reconnection in the heliosheath, with multiple Sweet-Parker reconnection regions.

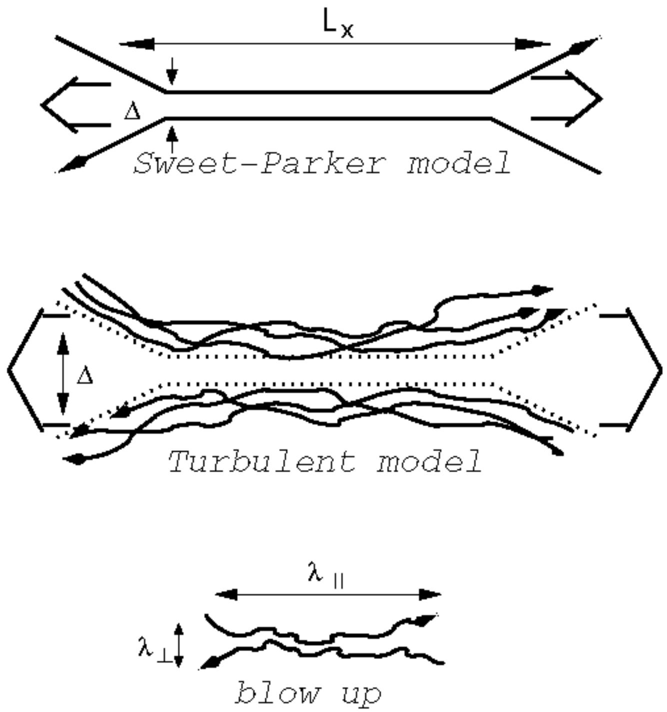


Figure 4. Upper plot: Sweet–Parker model of reconnection. The outflow is limited by a thin slot Δ, which is determined by Ohmic diffusivity. The other scale is an astrophysical scale $L \gg \Delta$. Middle plot: reconnection of a weakly stochastic magnetic field according to LV99. The model that accounts for the stochasticity of magnetic field lines. The outflow is limited by the diffusion of magnetic field lines, which depends on field line stochasticity. Low plot: an individual small-scale reconnection region. The reconnection over small patches of the magnetic field determines the local reconnection rate. The global reconnection rate is substantially larger as many independent patches come together (from Lazarian et al. 2004).

Drake, et al, 2009 had a different proposal which also involved reconnection:

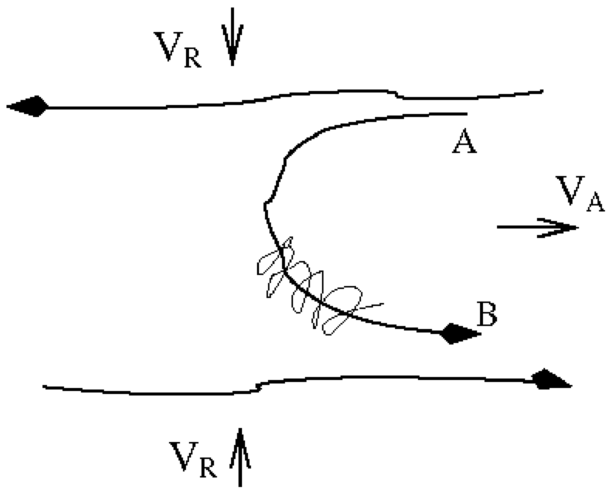


Figure 6. Cosmic rays spiral about a reconnected magnetic field line and bounce back at points A and B. The reconnected regions move toward each other with the reconnection velocity V_R. The advection of cosmic rays entrained on magnetic field lines happens at the outflow velocity, which is in most cases of the order of V_A. Bouncing at points A and B happens because of either streaming instability induced by energetic particles or magnetic turbulence in the reconnection region. In reality, the outflow region gets filled in by the oppositely moving tubes of reconnected flux (see Figure 5) that collide only to repeat on a smaller scale the pattern of the larger scale reconnection. Thus, our figure also illustrates the particle acceleration taking place at smaller scales (from Lazarian 2005).

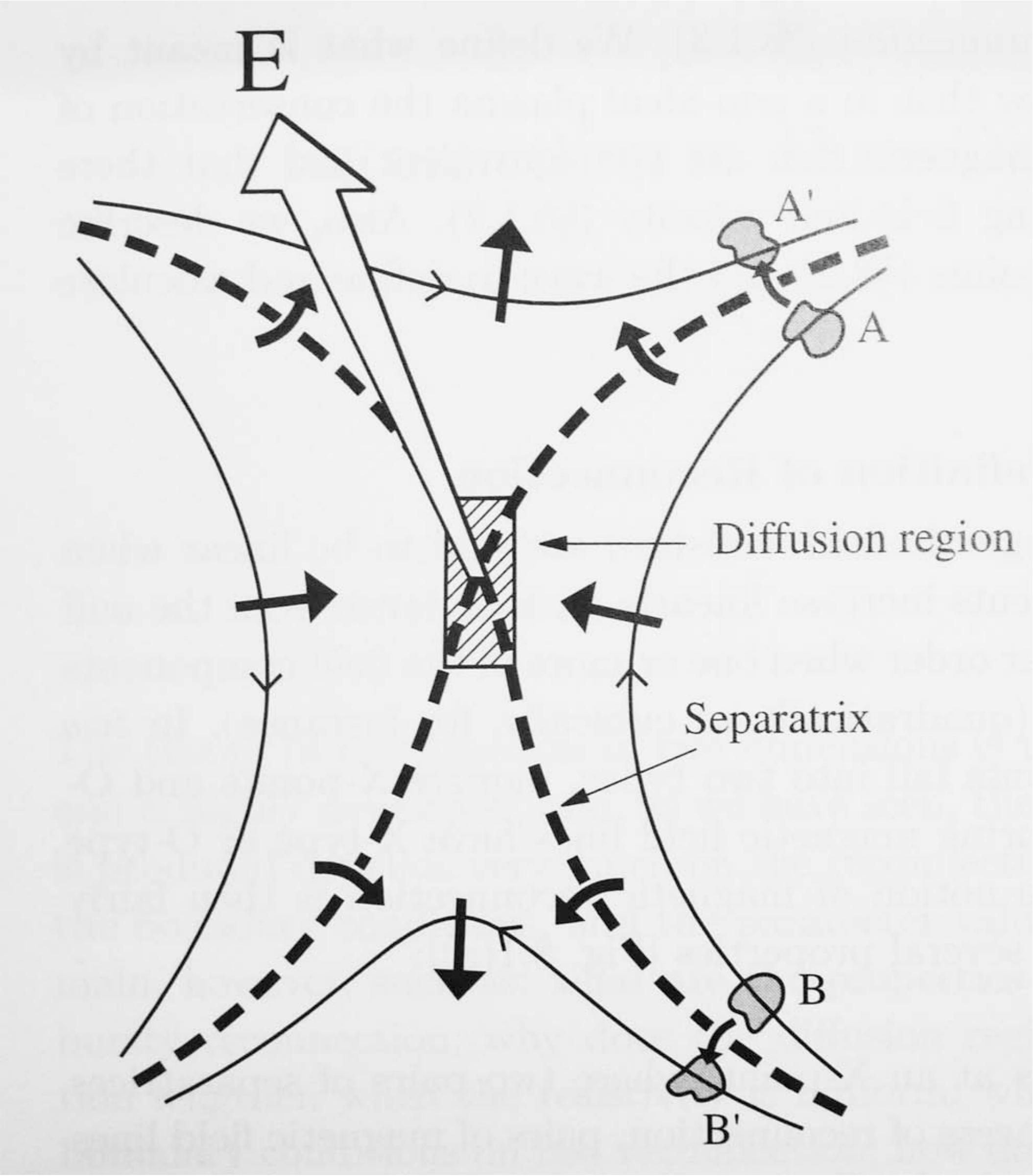
Analysis of Acceleration in Reconnection

- Recent papers (e.g. Lazarian and Opher, 2009, and Drake, et al, 2009) have proposed acceleration at reconnection events in the heliosheath, based on 2-dimensional simulations. ACR cannot be accelerated in a single event, as suggested by Lazerian and Opher.
- The electric field **E = -U x B/**c is normal to the frame of the simulation. |**U**| is about the Alfvén speed in the heliosheath, which is significantly less than the solar wind speed. Hence the electric field is significantly smaller.
- Hence, even if the scale of the reconnection event is the scale of the heliosphere, a *single reconnection event* cannot yield the 200 MeV/charge ACR.

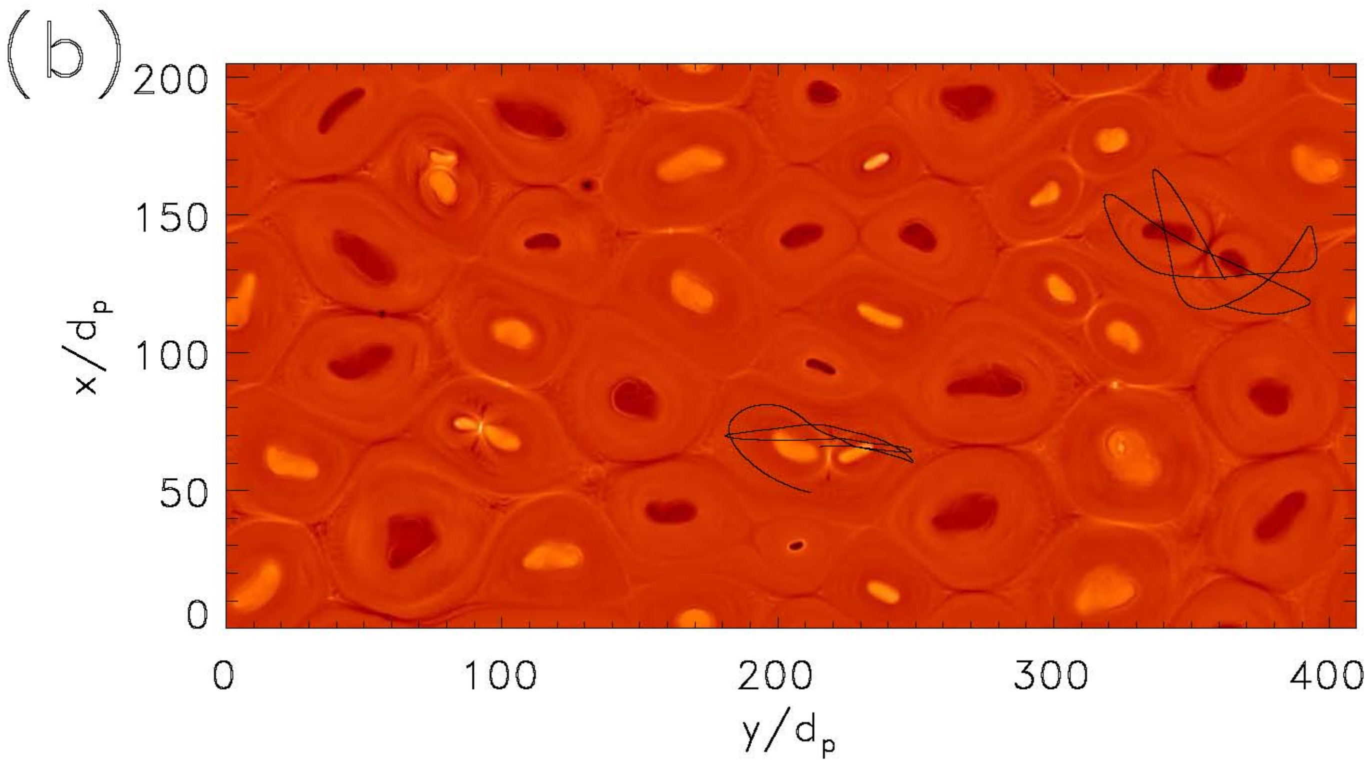
Consider the electric field.

To gain energy, the particles must drift in the direction of the electric field.

Since the flow speed is V_a, which is significanlty less than the solar wind speed, the required spatial scale is larger than the scale of the heliosphere. This is not likely.



- Drake, et al suggest multiple, coalescing reconnection ‘islands’, which may not be subject to the length-scale argument.
- Th time to accelerate must be considered. If particles gain of order $\Delta T = q\, r_g\, E = q\, r_g\, V_a\, B/c$ in each interaction, and each interaction takes a gyroperiod $2\,\pi/\omega_g$, then acceleration of oxygen to 200 MeV takes about a year, which is fine.
- So, multiple consective reconnection islands, with no time in between islands satisfies the primary constraints.


(b)
200
150
100
50
0
x/d_p
0
100
200
300
400
y/d_p

Next, consider the Parker Transport Equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i}\left[\kappa_{ij}^{(S)}\frac{\partial f}{\partial x_j}\right] \quad \Rightarrow \text{ Diffusion}$$

$$-\mathbf{U} \cdot \nabla f \quad \Rightarrow \text{ Convection w. plasma}$$

$$-\mathbf{V_d} \cdot \nabla f \quad \Rightarrow \text{ Grad \& Curvature Drift}$$

$$+\frac{1}{3}\nabla \cdot \mathbf{U}\left[\frac{\partial f}{\partial \ell np}\right] \quad \Rightarrow \text{ Energy change}$$

$$+Q \quad \Rightarrow \text{ Source}$$

Where the drift velocity V_d due to the large scale curvature and gradient of the average magnetic field is:

$$\mathbf{V_d} = \frac{pcw}{3q}\nabla \times \left[\frac{\mathbf{B}}{B^2}\right]$$

- Most models of reconnection published to date use incompressible MHD. Hence $\nabla \cdot \mathbf{U} = 0$. (
- Parker's equation only accelerates particles if $\nabla \cdot \mathbf{U}$ is finite and negative.
- Since Parker's equation has been shown to be valid for nearly-isotropic particle angular distributions, acceleration in reconnection models requires *significant anisotropies* pointed out by Drake, et al Ap. J. 709, 963, 2010).
- This may be difficult to do, because scattering times are generally shorter than the acceleration times. We need $\tau_{scat} \approx$ the acceleration time, or one year. The mean free path is >3000 AU Note to observers: look for large anisotropies.

Summary and Conclusions

- The frequently observed v^{-5} observed superthermal particle spectrum is important and needs explanation.

- Compressive-diffusion acceleration may be a solution. The equation must have a specific form to conserve particles. Fisk-Gloeckler 2008 does not conserve.

- Charged-particle acceleration in the heliosphere, especially of the ACR remains difficult. The termination shock can do it.
- Statistical acceleration is too slow.
- Reconnection has been proposed, but issues of the large anisotropies and mean free paths must be resolved.
- The ACR can be accelerated at the termination shock, if the acceleration has hot spots which the Voyagers missed.